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ABSTRACT

Children have many natural opportunities to learn about the basic aspects of quantity. This booklet is addressed to parents who want to support their children's mathematical growth. The activities presented suggest many ways in which parents and children can use mathematics in their environment. The activities are organized around common activities like shopping, gardening, or riding in the family car. Each chapter covers many mathematical concepts. Chapters are: (1) "Family History"; (2) "Personal Statistics"; (3) "Kitchen"; (4) "Laundry"; (5) "Water"; (6) "Gardening"; (7) "Telephone"; (8) "Grocery Shopping"; (9) "Post Office"; (10) "Restaurants"; (11) "Cars--Travel"; (12) "Money Management"; (13) "Hobbies"; (14) "Weather"; and (15) "Time." Also included are notes on mathematical concepts, and computation examples. Appendices include lists of books on mathematics, games and gifts, resources, a formula for use in finding the day of the week, palindromes, and discussions of intuitive geometry for preschoolers. (CW)

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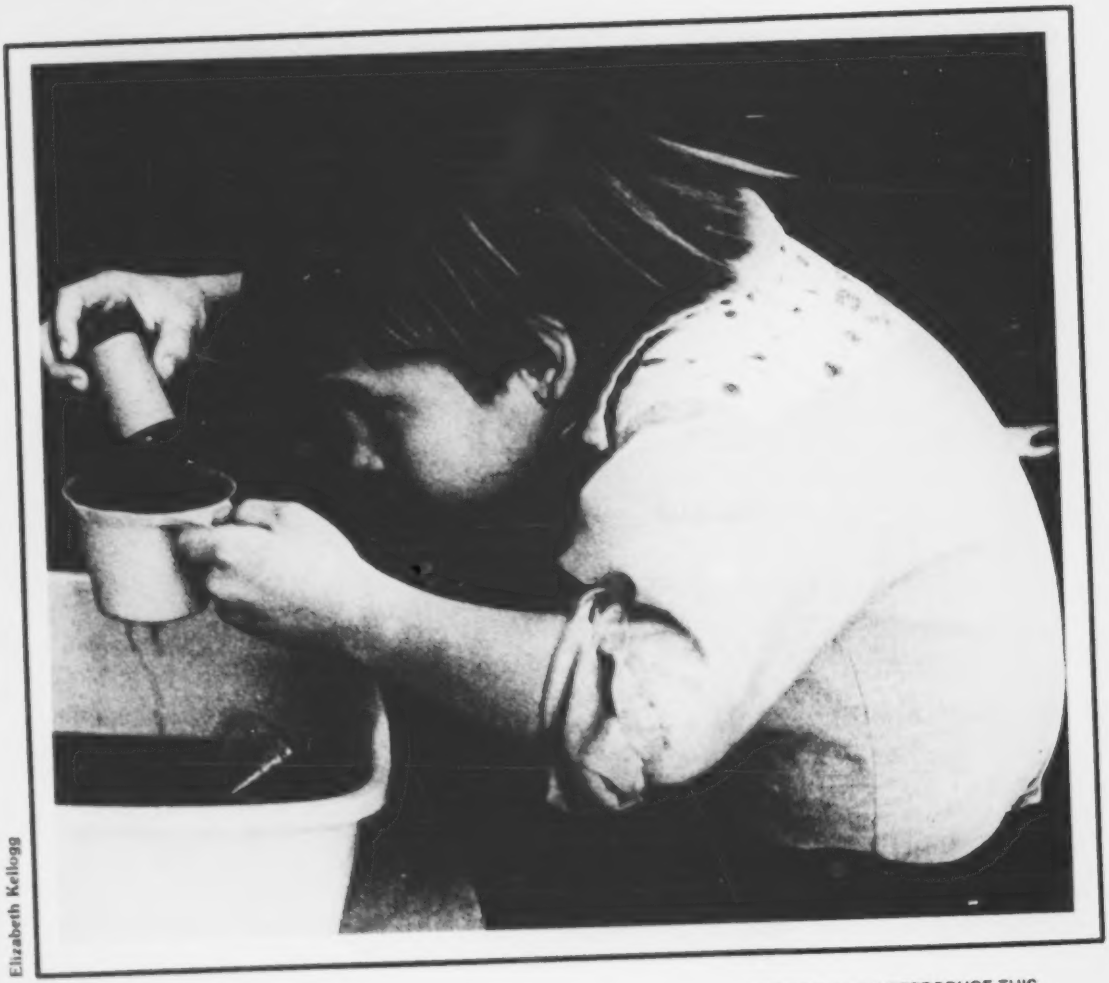
Pizzas, Pennies and Pumpkin Seeds

Mathematical Activities for Parents and Children



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Pizzas, Pennies and Pumpkin Seeds

Mathematical Activities for Parents and Children

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I wish to thank Tony Kallet for his careful reading of the manuscript and his excellent editorial advice. I welcomed his enthusiasm for the content of the book and his suggestions for clarifying the text. I am sad to say that this is the last manuscript of mine that Tony read. He died unexpectedly in April 1989.

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Maja Apelman
Boulder, Colorado
June 1989

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NOTES TO PARENTS

. . . it is hard to see how **any** child, rich or poor, Western or non-Western, can grow up in an environment which does not offer him or her the **natural opportunity** to learn about the basic aspects of quantity. In this sense, environments are all similar and quantity is universally available.

Herbert Ginsburg in ***Children's Arithmetic***[1]

This booklet is addressed to parents who want to support their children's mathematical growth. The activities presented here suggest many ways in which parents and children can use mathematics in their environment. By bringing mathematics close to home, to familiar activities which children enjoy, we hope to stimulate a new interest in a subject that is often seen as difficult or irrelevant.

You don't need any special skills or knowledge for the mathematical activities presented in this booklet. You do, however, have to become involved in mathematics yourself if you want your children to become interested and to learn. It's best to approach the content of this booklet with a spirit of adventure and curiosity. We are not asking you to **teach** mathematics to your children; we are asking you to **explore** mathematics **with** them.

Try to focus for a few moments on the mathematics in your immediate environment. Look around you or think about what you have done since you got up this morning. Can you see or remember anything mathematical? Did you look into a mirror? Reflection and symmetry are best described mathematically. Did you water your flowers? Flower petals are arranged in orderly mathematical patterns. How many different shapes can you see from where you are sitting as you read this? Geometry deals with the properties of these shapes. Can you see any shadows? You can predict the length of a shadow from the angle of the sun's rays or a lamp's beam. When you estimate what size bowl to choose for your left over rice, when you measure for curtains or look at the temperature on your outdoor thermometer, when you decide if the weather is safe for a cookout, you are dealing with some aspect of mathematics.

[1]***Children's Arithmetic*** by Herbert Ginsburg, Pro-Ed, 5341 Industrial Oaks Blvd., Austin, TX 78735, p.31.

Many people think primarily about arithmetic when elementary school mathematics is under discussion, but there is more to mathematics than balancing your checkbook or figuring your income tax. Geometry, estimation, measurement, patterns, statistics, probability and logical reasoning all have their place in the elementary school math curriculum. In this booklet we have gone beyond arithmetic and have included activities dealing with all of the above topics.

Unlike teachers in often crowded classrooms, parents don't have to worry about covering a prescribed curriculum. You can use whatever mathematical topics interest your children and give them some challenging problems. You can also help your children make connections between the more formal math they are learning in school and the math of their own everyday world. Many children need help to understand how their school math relates to the mathematics they do out of school. Parents can play an important role in bringing home and school math closer together.

We have given you many ideas and have asked many questions in the hope that you will catch the spirit of inquiry that runs through this book. Although we are recommending many mathematical activities for your children, we are not trying to give you recipes or prescriptions. Rather, we are trying to show you a way of thinking about everyday mathematics, a way of raising questions and of seeking answers, a way of trying out different approaches — in short a way of using your own creativity and that of your children to find and solve many additional mathematical problems. We would be happy to know that you tried and enjoyed some of the suggested activities. We would be even happier if our approach inspired you to continue looking for mathematics in your environment.

Mathematics is the language of science. Have you ever noticed the delicate curves in tall grasses

caused by the weight of their ripening seeds? A mathematician can describe this curve with a formula so that another mathematician could draw an exact replica of the curve without ever seeing the grass.

Mathematics can describe things with numbers and shapes that are hard to describe with words. For instance: the shape of the arc made by a ball flying through the air; the accelerating speed of a falling object; the turbulence resulting when two bodies of water meet; mathematical formulas can tell you how the honeycomb in a beehive is constructed, how billiard balls travel on a table top, how knots are formed or how the spiral of the chambered nautilus shell grows to accommodate the growing animal within.

These are exciting mysteries which one can enjoy even without understanding all the mathematics involved. Perhaps some of your children will want to stay with mathematics in order to gain a deeper understanding.

How to Use This Book

In order to show how mathematics is part of almost every aspect of your life, we have organized this book around common activities like shopping, gardening or riding in the family car. Each chapter covers many different mathematical concepts. Chapter headings tell you what activity the chapter is about. Most chapter subheadings refer to the mathematical content.

The activities in the book do not follow any special sequence. You can start anywhere you want. Look at the table of contents, go to the chapters that interest you most and feel free to skip around.

There are no references to specific ages. Some activities are clearly suitable for very young children; others will require the maturity of upper elementary students. Many activities, however, will interest children of all ages and can be approached at different levels of understanding.

Throughout the text, whenever we thought that you might have a question about mathematical terminology or methods of computation, we referred you to one of the two mathematical chapters ***Mathematical Notes*** and ***Examples of Computation*** at the end of the book. We hope they will be helpful.

Since we tried to give you many ideas, you will find a large number of activities listed in the text. We don't expect you to jump right in and try all of these activities. That could be a full time job for several months! Look at the text like a menu from which you can choose whenever you have some free time or feel like doing some math with your kids.

We have also asked you to become more aware of the mathematics in your and your children's environment, but we know that you will not want to "think math" all of the time. If you did, your children might soon tire of listening to you. Our advice is: Go easy! Think about math when it gives you pleasure and don't make yourself do it or force it on your kids when they are thinking about something else. Enjoy yourself, be playful, have fun with your kids and use any approach to solving problems that works.

An old African proverb says: "Give me a fish and you feed me for a day; teach me how to fish and you feed me for life." We have tried to show you **how** to fish in the hope that your children will benefit from your newly acquired interest in mathematics.

ACTIVITIES

ABOUT YOURSELF

1. FAMILY HISTORY

Each family has its own unique history, and most children are fascinated with stories from their parents' childhood and their grandparents' lives. It takes young children many years to develop a sense of time. At first, the near and the distant past are quite undifferentiated. What does "long ago" mean to first graders? They might say: "A long, long time ago, when my daddy was a little boy ..." or, "A long, long time ago, when Christopher Columbus discovered America ..." or "A long, long time ago when there were dinosaurs walking around ..." Historic dates are meaningless and even ten or twenty years represent a time span beyond children's comprehension. It helps, however, to approach the past through familiar people and events and to move back from the present gradually, in small increments of time. Children like to look at old family photographs, try on old clothes, and see how old tools and kitchen utensils worked. If these objects can be associated with the near past — "ten years ago when your big brother was a little baby ... twenty-five years ago when your mommy went to first grade ... fifty years ago when grandma and grandpa got married ..." it will help children to develop a sense of time.

THEN AND NOW

For elementary school children, their own past is history, even if it consists of only five or ten years. Children like to measure their growth by comparing the present with their past. A first grade teacher I know gave her children long strips of paper, divided into three sections. She headed the sections as follows: "When I was a baby ..." "When I was little ..." "Now ..." The children made drawings and wrote or dictated their text. They talked about crying and sleeping a lot as babies, sucking their thumbs and playing in the sandbox when they were little, going to school and riding their bikes "now." If your children are older, you might get a wide roll of adding machine tape and help them to divide it into sections representing the different years of their life. See what they can remember, help them remember more, and encourage them to draw or write about important events in their lives. Such personal time lines are a good starting place for developing an understanding of history.

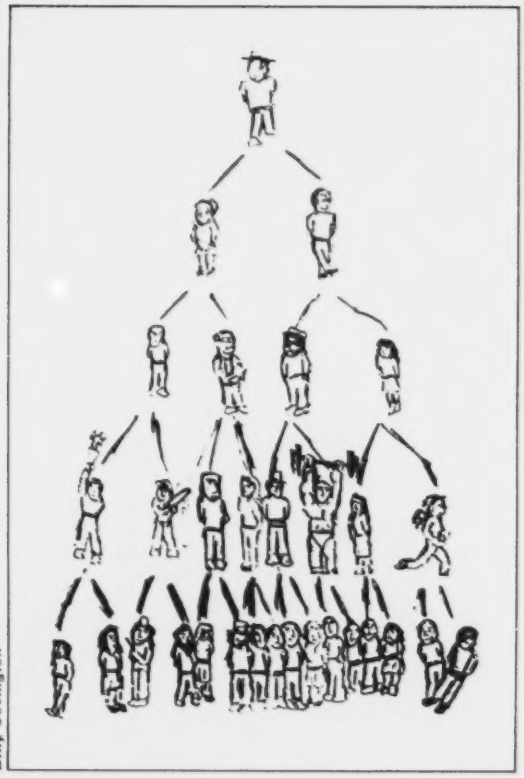
There are many comparisons that can be made with the past. (See also ***Personal Statistics***.) "How many hours did you sleep when you were a baby and how many hours do you sleep now?" Upper elementary children could convert these hours into percentages of a twenty-four hour day. (See ***Examples of Computations***.) If you still have some of your children's old clothes, let them compare these with the clothes they are wearing now. This would be especially interesting if you can compare the same type of clothes, like jeans, dresses or polo shirts. In mathematics, objects that are different in size but have the same shape are called **similar**. Similarity is an important concept in geometry.

There are "then and now" facts which older children could research: how large were families fifty or a hundred years ago and how large is the average family today? How long did people live in past years and how long can you expect to live today? How tall were your ancestors as compared to people living now? Once you start your kids off with some ideas, they will enjoy adding their own.

HOW MANY GREAT-GREAT-GRANDPARENTS?

Doubling is a useful skill to have when you learn to add numbers in school. It allows you to make shortcuts: if you know that 2+2 is 4, it's not hard to figure out that 3+2 or 2+3 must be 5. Here is a doubling problem which your children will enjoy:

If you wanted to make a picture of five generations in your family — you, your parents, your grandparents, your great-grandparents and your great-great-grandparents, how many people would you have to draw? One child has two parents (other than step parents), the two parents each have two parents which amounts to four grandparents; they have two parents each, and so on. By doubling the members of each successive generation, you quickly get to rather large numbers. There are 4 grandparents, 8 great-grand-



Billy Covington

parents, 16 great-great-grandparents and so on. Five generations add up to 31 people. How many people would there be in ten generations? (1023) In the above calculations, I have counted only the child's direct ancestors. Imagine how large a family reunion of just three or four generations would be if you included brothers and sisters, aunts, uncles and cousins, as well as all the in-laws!

Give your children calculators to perform the above doubling computations. (See ***Using Calculators***.) Some children will be satisfied to figure out the number of people in just four or five generations, others would enjoy going back further and seeing how fast numbers grow when doubled over and over again. This sort of playing around with numbers and number patterns really fascinates some children. Encourage this playfulness — it is a legitimate aspect of doing mathematics.

HOW OLD WOULD YOUR GREAT-GRANDPARENTS BE IF THEY WERE ALIVE TODAY?

After figuring out how many more people there would be in each successive generation, encourage your children to estimate how many years back they have to go to get to their grandparents' or great-grandparents' childhood. Taking 25 years for one generation, the birth of their great-grandparents would have been about one hundred years before their own birth. Old photographs and other mementos of past times would help to give these numbers meaning.

If you know the actual birthdates of great-grandparents and other family members of past generations you could calculate how old all these family members would be if they were still alive. This would give your kids good addition and subtraction practice.

HOW MANY GENERATIONS?

On a recent plane flight, my collaborator sat next to a woman who was traveling with her baby daughter. They were on their way to a family reunion at which the woman's great-grandparents were going to be present. How many generations of this family will be at the reunion? What's the largest number of generations that could be alive at the same time?

You might well ask: what is the point of all this? It won't interest all children, to be sure, but figuring out how old **your** great-grandmother or great-great-grandfather would be if she or he were still alive, is a personal way of thinking about "old" and "long ago" which helps children to a better

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Family Reunion

understanding of time. The photographs of wonderfully fit-looking 100+ year old folks which Willard Scott likes to show on the ***Today Show*** certainly bring meaning to birthdates in the 1880's.

Numbers don't have to be dry. They can tell stories and open doors to a fascinating past.

2. PERSONAL STATISTICS

Children are always interested in facts and figures dealing with themselves. "How old am I? How tall am I?" "How many teeth have I got?" "How fast can I run?" — these are just a few examples of the many things that can be counted or measured. You can give your children a special book in which they can record their personal numbers and statistics.

HEIGHT AND WEIGHT

Numbers are part of our lives from the moment of birth. A newborn baby's length and weight are measured and both of these statistics are entered on the hospital's birth record together with the date and time of birth. A few minutes after you were born, you had four sets of uniquely personal numbers: your length, your weight, and the date and time of your birth. (When does "length" change to "height"? When a toddler can stand upright?)

Length and weight are measured periodically when parents take their children for check-ups. How long did it take for your child's birth weight and length to double? If you have kept any of these records, share them with your children; if not, ask your pediatrician if he still has these records on file. Children love to know all about their beginnings and there are many arithmetical computations they can do using these records. For instance: How much have they grown since birth both in height and weight? How many times as tall or heavy are they now? What was their growth rate at different times in their lives? Half an inch a month? Four inches a year? When did they grow the fastest? What was the average for their life span?

If children are studying percentages in school, ask them to calculate the percentage increase of their current height and weight over their birth height and weight. (See ***Examples of Computations***.) If your children want to use a hand calculator for their computations — and there is no reason why they shouldn't — they will have to

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change any fractions into decimals. (Again, see ***Examples of Computations***.)

Here are a few more length and weight activities: It's nice to keep a record of your child's growth. You can record your children's height on a wall or on a piece of paper attached to the wall. Measure their height at regular intervals and write down the date next to the line indicating the height. Marking down the children's heights in both U.S. and metric measurements would help them to become familiar with both these standards. Metric rulers and tape measures can be purchased in many variety and department stores.

Some bathroom scales are marked in both pounds and kilograms. It's more fun though to have two separate scales — if possible. Your children can weigh themselves on the two scales, compare their weight in the different units, and begin to get a feeling of weight as expressed both in pounds and in kilograms. What about weighing yourself before and after you eat, or just after you get up in the morning and again after your evening meal? Children could also weigh themselves with and without clothes, or compare the weight of their winter and summer clothes.

Have you ever tried weighing something on your bathroom scale that wouldn't fit — like a

large turkey roast, an odd-sized package or a wiggling pet that won't sit still? Here is a simple trick: Ask your child to get on the scale holding the object you want to weigh. Record the weight, then weigh your child without the object. One simple subtraction will give you the correct weight.

Not all children of course will want to do all these calculations. My purpose in listing them is to show how much mathematics can come out of playing around with just two personal measurements: height and weight. The more you think about a subject, the more ideas you will get.

PERSONAL NUMBERS AND MEASUREMENTS

Following are some more ideas for personal statistics which your children can record in their special number books.

First, some **personal numbers:**

- Birthday[1]
- Year of birth
- Age
- Address, including zip code
- Phone number, including area code
- License number of your car
- Social Security number
- Number of baby teeth lost, with dates
- Number of permanent teeth growing in, with dates they broke through gum.

Next, a list of possible **body measurements**:

Measure the length of:
fingers, hand (longest dimension), arm, leg, foot, handspan and armspan.[2] Compare right and left measurements wherever applicable. Are there differences?

Measure the width of:
your hand, your eyes, your mouth — in repose and smiling broadly.

Measure circumference of (measure around):
fingers, wrist, upper arm (hanging down and held up with muscle flexed), neck, head, waist, chest, and hips.

Elizabeth Kellogg

You could talk about different sizes of clothing when you are taking these measurements and show your children the body measurement charts on sewing patterns or in catalogs. Here are some questions your children can try to answer:

- What is the origin of sizes for hats and gloves, for shoes and socks?
- Are ring sizes related to gloves sizes?
- What does 15/32 mean in a man's shirt?
- What does 34/30 mean in a pair of man's jeans?
- Why aren't shoe and sock sizes the same?

When I was a child, I learned that if the foot part of a sock would fit around my fist, the sock was the right size. That suggests a relationship between the size of hands and feet. You can have your kids check that out with different family members.

Children who are studying **ratios** in school can do some interesting calculations with body measurements. (see ***Examples of Computations***.)

– Are you a "square" person? Measure your height and the width of your armspans. Are they about the same? Then you are a square! Some people are tall or wide rectangles.

[1]See Appendix for a formula with which you can find the day of the week of any date between January 1, 1800 and December 31, 1999.

[2]Children might be interested in knowing that the yard was said to be established as a unit by King Henry I: he measured the distance between the tip of his nose and the tip of the finger of his oustretched arm.

– Which part of your arm is longer, from your elbow to your shoulder or from your elbow to your wrist? What if you include your hand? What is the ratio between upper and lower arm? Can you do similar calculations for your leg, measuring up and down from your knee?

– In a number of African countries, children are considered ready for school if they can reach over the top of their head with an arm and touch the opposite ear. (The head-arm ratio changes as they grow.) Can your five-year-old do this?

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– With a soft tape measure, take the following measurements: around the base of your thumb, around your wrist, around your neck and around your waist. My measurements in inches for these body parts are: 2 3/4, 6, 12, and 26. Each measurement roughly doubles the previous one. Young children who have not yet learned to measure with standard units like inches or centimeters could use different pieces of string for each measurement. These pieces could be put in order from the shortest to the longest, and then compared. If you paste or staple the pieces of string onto some cardboard and label them, you will have a simple graph.

– An old saying goes: "Once around the neck: twice around the wrist. Once around the waist: twice around the neck." Is this true?

THREE WAYS TO MEASURE YOUR HAND

Sometimes there are different ways to measure one thing. How many ways can you measure your hand? You can measure the **length**, from wrist to tip of longest finger, and also the width. You can measure the **area** of your hand: place the hand that you do not use for writing on a sheet of graph paper, palm down and fingers open, and trace around it with a pencil. Then count the squares. Skip the squares that are smaller than half, and count those that are larger than half as full squares. If you are using centimeter graph paper, the count of the squares will give you the area of your hand in square centimeters (cm^2). If you are using quarter inch graph paper, outline a one-inch square and you will see that it contains sixteen quarter-inch squares. If you divide the number of squares covered by sixteen, you will get the area covered by your hand in square inches.

You can also measure your hand's **volume** (the amount of space it takes up) by seeing how much water your fist displaces. Put some water in a glass jar and mark the water line with tape or marker. Submerge your fist into the water up to your wrist and record the new water line. The amount of water between the two lines is a measure of the volume of your fist. You could pour this amount into a measuring cup to see your fist's volume converted into liquid ounces or mililiters. Finally you could measure how much your hand can hold. Fill a bowl with small beans — soy beans work well — and see how many you can hold in each hand. Are the right and left hand the same size? How do these three hand measurements compare?

ADDITIONAL ACTIVITIES

Here are some more number activities for children:

How many times can you bounce a ball, shoot consecutive baskets, jump rope?

How many steps do you climb in one day?

How many steps does it take you to walk around your block? to walk half a mile?

How fast can you walk, run, skip, bike a given distance?

How high can you reach or jump?

How long can you hum one note, stand on one foot (with eyes open and with eyes closed), hold up a heavy object with one arm?

How much can you expand your chest? After your kids exhale as much air as they can, take their chest measurement; then have them inhale as much as they can and take another measurement. The air in their lungs accounts for the difference.

And finally some measurements of the heart:

What is your heart rate? Show your children how to take their pulse or do it for them. The big artery on the side of the neck is much easier to use than the inside of the wrist, which doctors prefer. How many times does their heart beat in a minute when they are resting; how many times after jumping up and down fifty times or running one block? With a calculator, your child could multiply her pulse rate to find out how many times her heart beats in an hour, a day, a week, a month, or a year!

This list of activities may seem long on first reading but it will grow even longer once you and your children start adding your own ideas. Mathematics is all around us and it becomes much more interesting when you begin to realize what role it plays in your personal life.

Billy Covington

IN YOUR HOME

3. KITCHEN

If you had to choose just one place in your home in which to support your child's mathematical learning, it would probably be the kitchen. Try to think of math for one day as you work in your kitchen and you will soon realize that mathematics is part of almost every activity.

You **estimate** when you decide how much spaghetti to boil for dinner or whether you have enough sugar left to bake some cookies; you **measure** quantities of ingredients, temperatures of oven or freezer, cooking time in the microwave; you **count** the number of plates, napkins, and glasses you put on the dinner table; you **compute** the roasting time of a twenty pound turkey; you **read numbers** on dials, timers, recipes and clocks; you work with **fractions** when you divide a pie or halve a recipe; you deal with three-dimensional **geometric shapes** like cones (funnels) and cylinders (cans of food); you estimate **volume** when you chose a container for left overs and you **create patterns** when you decorate a cake or arrange foods for special occasions. All such activities and many more can be shared with your children.

READING NUMBERS

Learning to read numbers is both a reading and a mathematical activity. If your pre-schoolers show an interest in deciphering numbers, be aware of opportunities in your house where they can practice this skill. In the kitchen, there are numbers on stoves, blenders, microwaves, toaster ovens, kitchen timers, clocks and more. Most of the numbers your children see in the kitchen relate to time or temperature. By talking about these numbers, children begin to see them in meaningful contexts in addition to learning to recognize and read them.

COUNTING

Young children love to show off their counting skills. This early rote counting does not indicate that children have grasped the full meaning of number. It means that they have memorized the words we have given to numbers and that they enjoy reciting them in order. Children at that stage can count along with actions like stirring, sifting, or mixing. Even if you have electric mixers, let your kids stir things by hand if you have the time and own some "old fashioned" implements like hand eggbeaters or sifters. Children can count, alone or with your help, as they turn the eggbeater or sift flour.

Elizabeth Kellogg

As children develop a deeper understanding of numbers they enjoy counting objects. At this stage, children love to count all kinds of things. Let them count the number of items you purchased at the supermarket when you unpack the groceries, the number of ingredients and utensils you put on the counter as you prepare to bake a cake, or the number of plates or glasses you remove from the dishwasher. Don't worry if children make mistakes, like skipping a number, or counting one object twice and another not at all, and don't try to correct them. Such mistakes are quite common when children are learning to count and only practice, time, and experience will teach them to do it correctly. Learning to count with understanding is a complicated process. (For further explanations see ***Learning to Count***.)

MATCHING

Matching is an important early math activity. (See ***Learning to Count***.) When children match one object with another object, they put them into "one-to-one correspondence": One object from one bunch of stuff is matched with one

object from another bunch. Let your youngsters match pots with lids, cups with saucers, knives with forks, or any sets of objects which you or they can think of. At meal time, there'll be one piece of pie for each family member; when setting the table, there should be one place mat by each chair, one napkin and one plate on each place mat, one glass by each plate, and so on.

There are many other opportunities for this kind of matching in everyday situations: while helping young children with dressing, parents can talk about one mitten for each hand, one sock and one shoe for each foot, one button for each buttonhole, two sleeves for two arms. Children also match things naturally in their own play: they cover each doll with a blanket, put a little person in each toy car, or take enough apples from a basket to have one for each friend. All the time they are practicing one-to-one correspondence.

In preschool, children have more experiences with matching: one box of crayons and one piece of paper for each child, one coat for each hook, one straw for each carton of milk. Lining up in pairs is a kind of matching, as is dividing up into groups — one child for the red group, one child for the blue group, one child for the green group.

In kindergarten, children are often given worksheets on which they are asked to match objects that belong together by connecting them with a line: for instance: rubber boots and umbrella, chicken and eggs, paint brush and easel, hammer and nails. This matching may not be one-to-one. Instead objects are matched because there is a logical connection between them. At home, children can practice this kind of matching and logical thinking with real objects as they themselves look for things that go together. This requires more active thinking and develops greater understanding than drawing lines between objects selected by someone else.

SHAPES

Everywhere in the world there are shapes. The most commonly seen shapes have been given names — circles, squares, rectangles, cubes, cylinders, spheres, and so on. These shapes, and many more, can be seen in city and country, in things made by people and in things found in nature. You'll also find lots of shapes in the kitchen.

There are many round shapes: plates and pot lids of all sizes, the tops and bottoms of pots and pans, different sizes of glasses and bowls — all examples of circles. There are many cylinders: most food cans as well as rolling pins, oatmeal boxes and rolls of paper towels. There are very small cubes, like sugar and bouillon cubes and ice "cubes" which aren't all cubes, and large cubes, like certain tins of tea. There are many boxes of different sizes — cereal boxes, boxes of crackers, of raisins and of sugar. Grapefruits, oranges, apples and grapes are spherical, though none are perfect spheres. Pears resemble tear drops, funnels and coffee filters are cones, strainers are hemispheres, and baking sheets and cutting boards are sometimes squares and rectangles.

The fact that a cylinder can be a tall can of tomato juice or a squat little can of tuna fish, with all possible variations of height and width in between will help children to see what it is that these cans have in common, what in fact makes them cylindrical. It is important that children see all these shapes in a variety of sizes.

Young children learn best through their senses. Handling shapes, feeling round or flat sides, edges and corners (these are called "lines" and "points" in geometry), seeing which shapes roll or spin, will help them remember what the different shapes are like. Children will remember the shape names more easily if they have had these kinds of first-hand experiences.

Let your children feel and play with the three-dimensional kitchen shapes — they will be way ahead when they meet them in geometry classes in school.

MULTIPLICATION

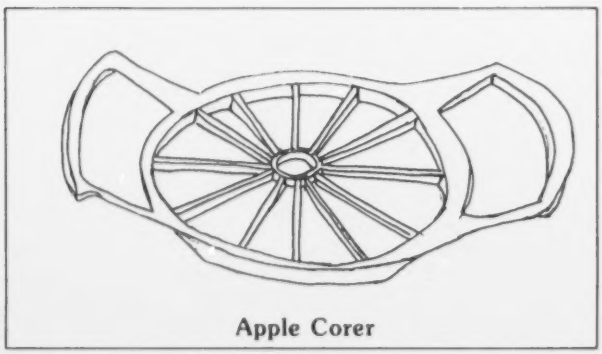
Anything that has rows and columns (i.e., rows in two directions) such as muffin tins or ice cube trays, can be used to practice multiplication. If you've baked some muffins, ask your child to count the number of muffins there are in a row. How many rows are there? How many muffins altogether? When you bake cookies let your kids help you place the cookies on the sheet and then have them figure out the total number by counting just one row and multiplying the number on that row by the number of rows on the sheet. Your children could also predict how many cookies will fit on a sheet after placing the first row.

Using a calculator, children can do some interesting multiplications. You probably know about how much milk, eggs or bread your family eats in a week. Let your kids figure out how much you use in a month, a year, or even in five years. They may want to do similar calculations with

other foods. It's fun to see your daily or weekly food consumption projected into such large numbers.

FRACTIONS

Often children learn about fractions in school with certain specially designed materials like fraction bars and they tend to associate **half** or a **quarter** with the shape with which they learned the concept. It's important that children see many different sizes and shapes of common fractions. The kitchen again provides such an opportunity. **Half** for instance, can be half a grapefruit, half a loaf of bread, half an avocado, half a stick of butter, even half a dozen eggs — and all look different. By being able to handle these different halves and putting them together again to see the original whole, children quickly learn to understand the meaning of **half**. The same holds true for other fractions.



Andre Cotto

Apple Corer

When you are dividing a pizza, a cake or a pie into separate pieces, or when you use an apple corer which also cuts the apple into eight or twelve equal sections, you are dealing with fractions as well as with geometry. Children could divide a banana into quarters, an orange, peeled or unpeeled, into fifths, a pie into sixths, or a pizza into sevenths or eighths. Let your children find their own strategies for dividing different shapes and let them also find out which fractions are relatively easy to section off and which are more difficult. These activities will help them with both arithmetic and geometry. (See also "Pizza Mathematics" in ***Restaurants***.)

A set of measuring spoons and cups would make a great gift when children are learning about fractions. When you are cooking, let them do as much measuring for you as possible. You could also provide them with a couple of bowls and some salt or sugar and encourage them to experiment with their own measuring utensils.

Many children have a hard time learning to compute with fractions. They can practice in the kitchen. How much is 2 x 1/3? — look at the measuring cup. What about 4 x 1/3? How much is 1/4 and 1/8 of a teaspoon? Fractions won't frighten kids if they learn about them in the kitchen, baking cookies or dividing their favorite pie.

CHANGING SIZE

Interesting changes can be observed when you soak, cook, or dry some foods. Soaking beans overnight in water changes their size dramatically. Whenever you soak beans, save some dry ones for later comparisons. Children can record this change by tracing around a dry bean and a soaked bean and comparing the size.

If you have a scale, children can also weigh the same quantity of dry and soaked beans? What accounts for the difference in size and weight? Is there some way in which you could measure the water that the beans have soaked up? Repeat this experiment with different beans and see if your kids can predict how much change there will be. If they speculate about what might happen before they do an experiment, they will be more interested in the results.

If dried beans grow large when soaked in water, will other foods shrink if you let them dry out? Try drying some apple or melon slices, mushrooms, grapes, or cabbage leaves by putting them on wax paper or stringing them up. The dry Colorado climate is ideal for such experiments. What accounts for the shrinkage? If you buy dried apricots or prunes and soak them in water, will they go back to their original size?

Experiments that draw attention to the ways in which foods change if water is added may make your children more aware of what happens to foods like pasta shells or rice when it is being cooked. Point out any such changes when you are serving foods that swell. Changing the size of foods through adding water is really a science experiment, but mathematics is the language of science: it's important that children learn how mathematics is used in the various sciences and how changes can often be better described with numbers than with words.

MEASURING TEMPERATURES

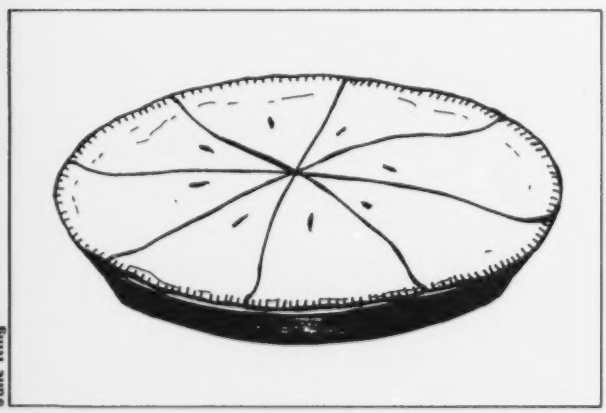
I once had a collection of thermometers used in the kitchen: an old-fashioned circular oven ther-

mometer, thermometers to measure the temperature in refrigerator, freezer, and in meat; and thermometers used in making candy, yogurt, and wine. Comparing different thermometers can raise interesting questions. How do they respond to heat? Some have a metal spring which moves a pointer, others have a narrow glass tube filled with colored alcohol. Both the metal spring and the colored alcohol expand when heated.

Let your children look at and compare the numbers on the thermometers. Why are they different for oven and freezer? Which has the highest numbers, which the lowest? Is the range of the numbers the same on the various thermometers? In addition to gaining valuable practice in learning to read numbers on a scale and in finding the "in between numbers" which are not marked, children will begin to think about degrees and temperature and why they are important. Knowing how to read numbers on a scale is a useful skill. Knowing what the numbers mean, however, brings interest and purpose to this skill.

DOUBLING AND HALVING RECIPES

Most of us change recipes occasionally to make more or less than the amount given in cook books. Increasing or decreasing ingredients in a recipe can raise interesting problems. If you double the ingredients for a cake, do you have to adjust the baking time? How do you select a pan of the proper size if you have doubled the ingredients? It would have to be twice as big but twice as big in which way? If you take a pan that has twice the diameter of the original pan, you can tell immediately by looking at it that it is much too large. How, then, do you select the proper size pan? If you want the cake to have the same thickness, the pan must have twice the area. (See p. 48 on how to calculate area.) If your kids like to bake cakes or cookies, ask them to halve or double the recipes and then let them experiment on their own to find out what adjustments they need to make. The results may not be quite right at first but they will learn a lot by the mistakes they may make.



Julie King

ESTIMATING VOLUME

Becoming familiar with the measurement of liquids takes experience. Liquids are sold in many different containers. Have your children start a collection for experiments with liquid volume. The different sizes of milk cartons will make a good start. You can buy cartons of milk in the following sizes:

one half pint	8 ounces or 1 cup
one pint	16 ounces or 2 cups
one quart	32 ounces or 4 cups
half a gallon	64 ounces or 2 quarts or 8 cups
one gallon	128 ounces or 4 quarts or 16 cups

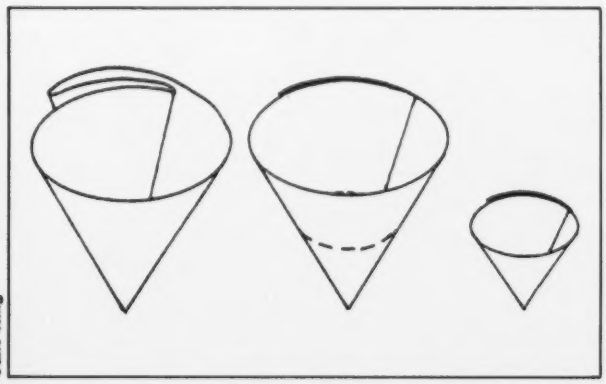
Note that the half pint, pint and quart container all have the same area at the bottom but they vary in height. The half gallon and quart containers have the same height but they vary in width. Add to the above collection a variety of yogurt and cottage cheese or margarine containers, any plastic storage containers or bowls of your own, as well as some funnels. Now let your children play and experiment with pouring water from one to the other. A few drops of food coloring will make it easier to see the water level in the transparent plastic containers. Children can do this at the kitchen sink, outdoors in the summer time, or in the bathtub.

Seeing how the same quantity of water can look different in containers of different sizes and shapes will help children towards an understanding of "invariance." (See footnote, page 15.)

How good are you at estimating which size bowl to select for some leftovers? I have a set of nesting bowls which range in capacity from three to twelve cups. The bottom half of each bowl fills up rather quickly but the wider top half always seems to hold more than I expect. Measuring the diameter of the top of the bowls, their different heights as well as their volume, I found much to my surprise, that a one-inch increase in diameter and half-an-inch increase in height accounted for a quart increase in capacity.

CONE EXPERIMENT

Take two round coffee filters or pieces of paper and shape them into cones, taping the edges together. Then cut one cone in half so it is half as high as the other. Now fill both cones with salt and then pour the contents, one cone at a time, into a measuring cup to find out how much salt each cone held. Did the larger cone hold twice as much salt as the smaller one or was there more than double the amount in the large cone? How much more? Were you surprised at the result? Look carefully at the cone: why is there so much more space in the upper half? Then look at your mixing bowls again. Does that help you to understand why your judgment can often be wrong when you are trying to decide how much more soup will fit into a bowl when it seems half full?



Julie King

In this experiment you were doubling the height of the cone. Height is one dimensional, it goes in one direction only. The empty space in the cone, however, increased in **volume** which is three-dimensional — height, width and depth. (See "Measuring Length, Area and Volume" in ***Measuring***.) There are mathematical formulas which describe and explain this relationship. For children in elementary school it is sufficient to notice what happens and to have many experiences with volume measurements.

4. LAUNDRY

Did you ever think that there was mathematics involved in doing your laundry? Laundry activities are particularly well suited to give young children experiences in sorting, classifying and matching — all of which are important for beginning number work.

SORTING

Before you do your wash you probably sort your laundry into several piles. In doing so you are creating different categories, classifying your laundry according to certain characteristics. Suppose you start out with a pile of mixed clothing which you then sort into two piles — one for the delicate cycle, the other one for regular wash. After that you may decide to divide up the regular wash into two further piles — dark colored and light colored clothing. You now have three piles, each one with different attributes: dark colors, light colors and delicate fabrics. On other days you might make different categories: your first two piles might be clothes that need to be washed right away and clothes that don't have to be washed right away or clothes which must be bleached and clothes which must not be bleached, and so on.

When you put away your dishes or your groceries, when you organize your sewing box or your collection of assorted nails and screws, when you separate your bills into "urgent" and "can

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wait" piles, you are sorting. You sort things all day long, possibly without even being aware of it.

In order to sort, you have to perceive likenesses and differences. That involves logical thinking: you have to notice how certain objects are alike and how they are different. Young children need time and experience before they can do this and before they can tell you by what kind of likenesses they have sorted something. The more experiences children have to sort simple things like buttons, crayons, or silverware by size, color, shape, or any other attribute they may think of, the more adept they will become in noticing and describing a variety of likenesses and differences.

In kindergarten or first grade children often play classification games. They enjoy classifying themselves: children with dark or blonde hair; children with long or short hair; children wearing sneakers, sandals or tie shoes; children wearing pants, skirts, or dresses; children wearing shirts with stripes and children wearing shirts with no stripes. Making classifications of this kind gives children excellent practice in logical thinking and in stating in their own words by what attributes they are sorting a mixed collection of things. Logic is the keyword here and logical thinking is absolutely basic to mathematical problem solving.

Apart from logical thinking, sorting activities with things or people prepare children for seeing differences in numbers. They may notice even and odd numbers and positive and negative numbers. They will learn to pick out multiples of three or five, as well as one, two or three digit numbers or numbers larger or smaller than ten. If they have worked with number patterns, they will also recognize number sequences like square numbers. (See ***Discovering Patterns***.)

If you have young children at home, let them help you with laundry sorting. You can tell your children what sort of piles you want them to make or you could simply say: "I never put colored and white things in the same wash" or "I always wash jeans separately" or "Nylons and other delicate fabrics have to be washed in a special cycle" and see if they can separate the wash according to your categories.

After the wash is finished, there is more sorting to be done. Maybe everything that came out of the dryer got thrown into a large basket. How are the clean clothes and linens going to be sorted? See what kinds of ideas your children have. They may sort by size — children's clothes and grown up clothes, by who the clothes belong to, by things that go in the kitchen and things that go in the bathroom. They may want to put all the socks, all the shorts, all the underwear and all the shirts into separate piles. There are endless possibilities.

Clearly you won't want to devote that much time to laundry sorting every week, but one or two sessions of multiple sorting will be well worthwhile. The longer children can sort and resort one set of stuff, the more carefully they have to look for likenesses and the more they have to think about what might go together. Once you get them started on sorting, they may want to sort all kinds of things in your house. For many kids sorting is a very satisfying activity.

MATCHING

Matching is discussed at greater length in the ***Kitchen*** chapter as well as in ***Learning to Count***. I am mentioning it here too because there is a lot of matching you can do in the laundry. Matching towels and washcloths; socks of the same size, color or pattern; tops and bottoms of two-piece outfits all reinforce the idea of one-to-one correspondence as it is explained in the section on ***Matching*** on p.9.

Here is a sock-matching riddle for older children: If there are ten red socks and ten white socks in your bureau drawer and it's dark in your room, how many socks would you have to take out to be sure you have a matched pair? (3)

OTHER LAUNDRY ACTIVITIES

In the process of helping with the laundry, children can also **practice counting:** how many socks did they put in the washing machine, how many socks did they match into pairs after the laundry was dried?

Can you let your child **measure out the detergent** to add to the wash water? Can you have different brands of detergent at home — liquid, powdered, and concentrated — so that your child can compare how much is used of each one?

If you take your wash to a laundromat, give your children some **money**, ask them to pick out the exact amount of change needed and let them put the coins in the machine.

Children can gain some experience with **time**: Can you teach your kids how to read and set the dials on the dryer? If they know how to tell time, they can figure out when the dryer will stop and

let you know, if you are busy elsewhere in the house.

If you hang out clothes on a wash line, your kids could time how long it takes to dry sheets or towels on a sunny, cloudy, or windy day. You can also do some **estimating**: how long is the wash line? How many shirts, pants, socks will fit on it? How many sheets? How much space can you save if you do not stretch out the sheets? and so on.

Even **geometry** plays a part in your laundry activities: how many different ways can you fold flat linens like sheets, towels, dishtowels, table

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cloths, and so on? Can you fold them into squares? into different rectangles? Making neatly folded piles of clean towels or sheets can be very satisfying and some children would enjoy experimenting with folding.

Here is another geometry question: If you iron napkins or handkerchiefs and press down the folds, can you predict how many different sections you will have before you open them up? A lot of spatial thinking goes into solving such problems.

5. WATER

If you own your home, do you have any idea how much water your family uses in a day, a month, or a year? Where do you use more water, indoors or out-of-doors? In this section, there will be suggestions for many measuring activities. It's always a good idea to ask your children to estimate first what they are about to measure. Estimating forces you to think about what you are doing. You may start out with wild guesses but as you get better at it, you learn to use and apply your knowledge to solve new and different problems. Estimating, problem solving, and logical thinking should all be part of measuring.

MEASURING WATER IN THE BATHROOM

Can you answer this simple question: what uses more water, a bath or a shower? Let your children figure out how to answer this question. If they say, "Well that depends on how much water you put in the tub or how long a shower you take," all the better. You want them to think! If they ask for help, you can suggest that they plug up the bath drain while they take a shower and record the water height on the side of the tub with a small piece of tape. Then have them run the water for a bath and again record the water level. Which used more water, shower or bath? How much difference was there? Your kids can take showers of varying length — one minute, three minutes, five minutes — and again mark the level of the water that has collected in the tub.

In order to know how many gallons of water you have used, you have to take some measurements. Fill a plastic gallon container with water and pour it into your tub. One gallon won't look like much.[1] Continue pouring water into the tub, one gallon at a time, keeping track of the number of gallons. When you reach the marks on the side of the tub, you will know how much water was used. This is a tedious way of measuring but it's worth doing one time.

How long does it take to fill the tub to a certain height? How much water flows out of bathroom faucets or shower head per minute? How many gallons of water do you use for a three-minute shower? How long does it take to run bath water for a ten-gallon bath, a fifteen-gallon bath? These are rate measurements which involve time. You are measuring gallons of water **per minute** which is similar to measuring number of miles **per hour**. In your bathroom, you would be measuring the rate of water flow.

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[1]Children under the age of six may have a hard time believing that there is as much water in the tub as there was in the gallon container even if they see you pouring it into the tub. It **looks** as if there is less water in the tub and this appearance deceives them. Educators say that children have not yet developed an understanding of "conservation" or "invariance" when they are misled by appearances. This understanding develops gradually. It cannot be hurried or taught.

Do you use a water-saving shower head to reduce the water flow? How much less water is used? A friend told me that he put such a shower head into the bathroom used by his teen-aged son who liked to take long showers. Water was saved, but not just because of the shower head: the reduced water flow apparently made standing in the shower less enjoyable, so his son's showers became much shorter!

After you have calculated the rate of water flow in your bathroom, you might compare it with the rate of flow at other faucets in your home. Are they all the same? If there is a difference, you can get into a discussion of **water pressure**. Why might the water pressure vary at different water outlets in your house?

Do you know how much water your toilet uses in one flush? (It's the amount in the tank.) Some people put a brick into the toilet tank to save water. Can your kids figure out why that works? Do you get sufficient water in your toilet flush with one brick in the tank? What about two bricks? How much water are you saving on each flush? How much in a day or a week?

Do you have a leaky faucet? Have you ever wondered how much water you are actually wasting? Put a measuring cup under a leaky faucet and have your children time how long it takes to fill up. Public Service says that one drop of water per second from a leaky faucet wastes two hundred gallons of water a month. Can your children check that out? A lot of calculations will be necessary to get from one cup to two hundred gallons and if your faucet leaks at a different rate, faster or slower than one drop per second, the calculations will be more complicated. Some children, however, will find it challenging to get the answer.

Activities such as these bring meaning to measurement problems which children have to solve in school. Volume measurements can be hard to understand without first-hand experience. It becomes easier and more interesting when children can measure things that are part of their everyday life.

ONE THOUSAND GALLONS OF WATER

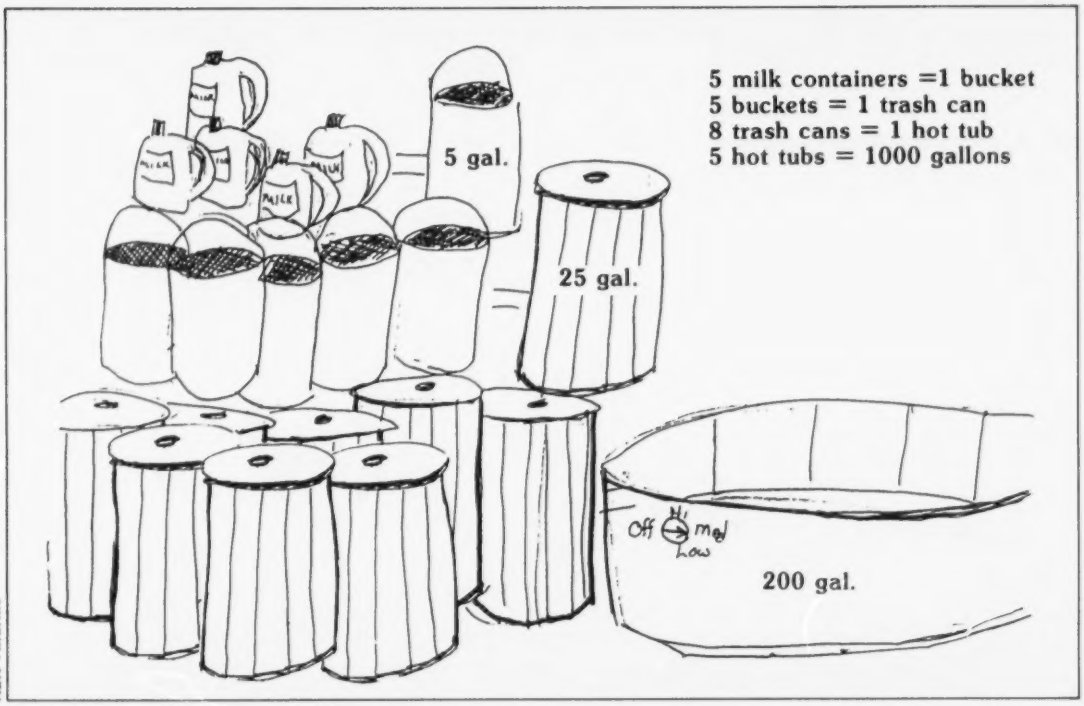
In many communities, water is metered: you pay for the amount you use in thousand gallon increments. (Some communities charge a flat rate, regardless of consumption.) Measuring water in the bathtub may help with an understanding of liquid volume for small quantities, but how do you gain an understanding of one thousand gallons? What sort of container would hold one thousand gallons of water? How many bathtubs full of water would you need to fill to get one thousand gallons? How many hours would you have to sprinkle your lawn to use a thousand gallons? How many washes would it take and how many times would you have to run a dishwasher to use one thousand gallons of water?

I have been paying my water bills for years without ever thinking about the amount of water I use. I realized that my bills were higher in the summer and that my water consumption increased when I had visitors, but I never tried to get a sense of the actual quantity of water that I was billed for until just now.

To **see** one thousand gallons of water in my mind, I started with one gallon, a known quantity. I filled a large bucket with water (I used a one-gallon plastic milk container) and found that the bucket held five gallons. Next I went to my trash can and filled that up with water from the bucket. I found out that my trash can held twenty-five gallons. Then I got stuck. I couldn't think of anything that holds approximately one hundred gallons — a capacity of four of my trash cans. A call to a hot tub dealer helped. A small hot tub holds two hundred gallons — eight of my trash cans. Five small hot tubs full of water took me to one thousand gallons.

This is one way of getting a feeling for a large quantity. To figure out when you have **used** one thousand gallons of water in your home is a little more complicated since all the water you use indoors goes down the drain as sewage. Here is a suggestion for a family project: try to measure how much water your household uses in an average day. Different family members could keep track of the amounts of water used throughout your house. Record the number of times the toilet is flushed, how many showers are taken, figure a way to measure the water used in kitchen and bathroom sinks, find out the capacity of your washing machine, and so on. If it's summertime, don't forget to measure the water coming out of your garden hose. (See ***Garden*** chapter for suggestion on how to measure outdoor water use.)

When you have found out how much water you use in one day, you can take the next step: how many days will it take you to use one thousand gallons? There may be many different ways of calculating this and you will be doing a lot of recording and problem solving. Do your cal-



Kate Andrias

culations agree with the amount of water used as recorded on your most recent water bill? These activities will also make you more aware of how you use water and how you could conserve it. Can you multiply your daily water consumption by the number of households in your community? How many thousands of gallons of water are used in one day? In one month? In one year? Many questions will arise as you and your children work on these problems.

One of the keys to real learning is curiosity. Asking your own questions and finding answers to them is not only satisfying, it gives children a sense of power as they gain control over their learning. They know they are becoming problem solvers, in mathematics as well as in life.

6. GARDENING

If you like gardening and enjoy spending time outdoors, you will find many opportunities to include some mathematics in your summer gardening activities.

PLANTING SEEDS

When I bought some Northrup King seeds I found that each package had the following little table printed on the outside.

- How many feet (or hills, in case of pumpkins) the packet will plant;
- How far apart to space the rows;
- How far apart the plants should be when they mature;
- How deep to plant the seeds;
- How many days till the seeds will germinate;
- How many days till the seeds can be harvested;
- A drawing of the seedling for identification;

In addition there are "growing tips" which tell you how thickly to scatter the seeds and when

and how much to thin. For carrots you are told to "scatter seeds at the rate of three to five seeds per inch. When seedlings have three leaves, thin to stand two inches apart." For lettuce, you are advised to "thin plants to ten inches apart to harvest entire heads. Thinning to four inches will yield a solid row of lettuce." And you are supposed to sow six to eight pumpkin seeds around a hill twelve inches in diameter and thin to three to four seedlings per hill after germination.

If you plan to grow a variety of vegetables, you can make a table that lists this information for all your plants. Here is an example of seven vegetables.

VEGETABLE	PACKET WILL PLANT	SPACING OF ROWS AND PLANTS	PLANTING DEPTH	DAYS TO GERMINATE	SEEDLING LOOKS LIKE	DAYS TO HARVEST
radish	20 ft.	R 10 in. P 1 in.	½ in.	4-6		23
lettuce	150 ft.	R 12 in. P 4-10 in.	¼ in.	7-10		45
pepper	45ft.	R 2 ft. P 18 in.	¼ in.	10-12		66
zucchini	10hills	hills 4 ft. P 3-4	1 in.	8-10		58
pumpkin	7hills	hills 8ft. rows 8ft.	1-1½ in.	8-10		100
carrot	40 ft.	R 1 ft. P 2in.	¼-½ in	14-25		73
beans	15 ft.	R 2ft P 4in	1½ in	7-10		65

Having the information organized in one table makes comparisons easy and can lead into many discussions and observations about the differences in plants. Have your children copy the drawing of the seedlings; this will help to motivate them to look for the tiny plants when they push through the earth. Note the differences between the leaf patterns of the seedlings — a nice example of the infinite variety of patterns in the natural world.

Now that you have all this information, what can you do with it? First of all, ask your children to help you with the planting. Have them measure your garden area and involve them in laying it out. Where will you put the different vegetables? How can you be sure you are planting enough for your family's needs but not so much that you will be looking for friends and neighbors to take cucumbers and zucchinis off your hands in the fall? Let your kids take the measurements for the proper spacing of rows. To make straight rows they should stretch some string from end to end making sure that the distance between the rows is the same at both ends. The strings will form parallel lines — an important concept in geometry.

After the seeds are put into the soil at the proper depth, there is nothing to do except water until they germinate. Note that radishes may push through after just four days while carrots may take up to twenty-five. Your kids could mark off these waiting days on a calendar or make a chart on which to record the number of days it takes for each species of seedlings to appear above the ground. Since children may not know that generally only a certain proportion of planted seeds will germinate, you can do some sprouting experiments before you plant your garden. It is a good way to use percentages and to introduce the subject of probability. (It's **possible** that all your seeds will germinate, but usually there are a few seeds in each bunch which won't.)

One simple way to sprout seeds is to line an aluminum pie or loaf pan with several layers of dampened paper towels. Spread at least ten seeds of one kind on the paper towel and cover the pan with saran wrap so that the towels will remain moist. (You may have to add some more water to keep the towels moist.) Ask your children to check the seeds every day and to write down when they begin to sprout.[1] How many seeds started to sprout after one day, after two days, after three days? and so on. How many seeds did not sprout at all? If six out of ten pumpkin seeds germinated, you can say that the **probability** that this bunch of pumpkin seeds will grow is 6/10 or 60%. If you get six plants from ten seeds, let your children figure out how many seeds they must place in the soil to get the number of pumpkin plants you want to harvest. Help them to see that 3/5 is the same as 6/10 or 9/15. To get three pumpkin plants you should plant five seeds; to get nine plants, fifteen seeds. The proportion in these examples remains the same as does the probability.

Your children may well ask why some species of seeds germinate sooner than others and why some seeds do not germinate at all. There are

[1] If you have a magnifying glass, encourage your kids to look at the seeds when the stem and tiny leaves first appear. It is a fascinating process that can't be observed when seeds are planted in soil since this early sprouting takes place under the ground.

some good science books for children on seeds in which they may find answers to these questions. Ask your local children's librarian to help you locate them.

After the seedlings have a few leaves and begin to look like the drawings on the seed packets the children can keep track of their growth rates. They can make predictions: how long before this plant is two inches tall, one foot tall? How high will it grow? How long will that take? and so on. The children's estimates can then be compared to measurements they take throughout the growing season. Again there will be some variety in the growth rates which will make taking measure-

from *Growing Seeds*, Elementary Science Study, Education Development Center, Inc., Newton, Mass. Used with permission.

ments more interesting. Older children can take measurements with standard units, such as inches and centimeters, but younger children can use strips of paper. (See ***Measuring***.) Place the paper strip alongside the plant, cut it to the length of the plant and staple or paste it to a piece of heavy paper or cardboard. The kids can decide how often they want to measure — every day, every other day, every week? It may depend on how fast the plant grows. In any case, dates of measuring should be recorded below the paper strips so that the rate of growth can be seen more easily — two centimeters per day, three inches per week, and so on. This is a very simple way of making a bar graph — one of the early types of graphs children learn to make in school.

If your children are interested, you could set up some experiments in the house at any time of the year. Beans (especially Chinese mung beans), lentils, grass seed, and corn all sprout quickly and grow quite fast. So do amaryllis once they come out of their bulbs.

Predicting, measuring, recording heights and dates, and organizing the information into a simple graph are all important mathematical activities. When they are learned in conjunction with planting and tending the family vegetable garden, the motivation is much different than when they are learned with textbook problems that may not relate to children's lives.

WATERING

You could also measure how much water each of your house plants can absorb at watering time, and how long it takes each plant to dry out. Does a big-leaved begonia need the same amount of water as a jade plant or a cactus? Some people water their plants at different times, others give them all the same treatment.

There is more garden related math:

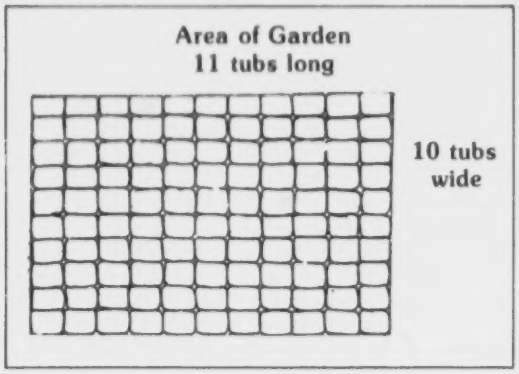
How much water do your yard and garden need? That, of course, depends partly on the weather — on temperature, humidity level, and wind which dries things out really fast. It also depends on the different plants: some need more water than others. We all know, however, that in a typical Colorado summer watering grounds and garden takes a good deal of time and a lot of water.

How much water **does** it take? You may know the answer in dollars or in thousands of gallons of water from your water bill but have you ever tried to figure out how much water you actually use when you sprinkle your backyard for an hour? Here is one possible way to do this. Get a rectangular plastic dish tub with flat bottom and place it in your yard or garden. Water for one hour. Measure how many inches or centimeters deep the water is in the tub after one hour. Assuming that your sprinkler rate is the same all over, you now know that you have watered the equivalent of say 3/4" of water in one hour.[1] That

[1]If you want to be more exact, put the tub in different locations at a number of different waterings and compare the amount of water that has accumulated in the tub. Then take an average of these various measurements.

is your watering rate. How do you translate that into number of gallons used for one hour of sprinkling?

First find out how many of your tubs it would take to cover your yard or garden. Encourage your children to come up with ideas on how to do this. Here is one suggestion: if the area is small,



you can count how many "tubs" long and wide it is and then draw a picture. In the illustration, each rectangle represents one tub and the area measures a total of 110 tubs — eleven multiplied by ten.

Next, pour the water that collected in your tub into a measuring cup. Say you have 5 1/2 or 5.5 cups of water. Multiply that by 110, the number of tubs needed to cover your garden, and you'll get 605 cups of water. Divide 605 by 16 to get the number of gallons — 37.8 in this case. (You divide by 16 because there are 16 cups in one gallon.) In this example, you use almost 38 gallons of water to sprinkle an area covered by 110 tubs. If each tub measured 8 by 10 inches, the area covered would be about 61 square feet which is a little less than a rectangle measuring 9 by 7 feet.[1] Can you estimate now how much water you might be using to sprinkle a much larger area? How often do you water your garden? How much water do you use in one month? Perhaps we would all become more careful in our summer water use if we did a few of these simple calculations.

[1]One 8 x 10 tub measures 80 square inches. (See ***Measuring Length, Area and Volume.***) 110 tubs measure 8800 square inches — 80 x 110. To convert square inches to square feet, you divide by 144 because there are 144 square inches in each square foot. 8800 square inches divided by 144 equals 61.1 square feet.

APPLYING FERTILIZER AND CUTTING GRASS

Here are two more opportunities to practice thinking about area: A bag of fertilizer will tell you what area can be covered and at what setting you should have your spreader. These and the previous computations on water use may well be too complicated for many children, but you can still let them know what you are doing. You might say: "This bag tells me that it contains enough fertilizer for 1000 square feet of grass. Let's see — how large is our lawn area; can you help me measure it?" After you do the measuring and multiplying you could say: "I should have about one quarter bag left over after I've done the fertilizing. Let's see whether it will work out that way." Including children in this way introduces them to the idea of measuring area without expecting them to understand all the steps or calculations. It is the best preparation they can have for the time when they will learn about area measurements in school.

This last example is for older children, probably teenagers. Do your kids earn money in the summer months cutting other people's lawns? How do they figure out how much to charge? Suggest to them that they charge by the area of grass they have to cut: a certain amount of money per square foot. Would that be a fair way? Would customers appreciate the precision of the charges?

A similar principle of charging could be used for shoveling snow. To figure out the charge, children would first measure the **area** to be shoveled — walk, driveway, sidewalk — and then set a **base rate**: a certain amount of money per square foot, the same way the charge for cutting grass was determined. This is the amount the customer would pay for one inch of snowfall. After each snowfall of more than one inch, your child must measure the depth of the snow, seven inches, for instance, and multiply that number by the base rate. The actual charge will vary with each snowfall and will reflect the volume of snow that was shoveled.

There are many ideas in this chapter for bringing mathematics into your garden. Once you begin to approach mathematics in the broad and playful way suggested in this booklet, you will undoubtedly come up with additional ideas of your own.

7. TELEPHONE

PHONE NUMBERS

As all parents know, young children are almost universally attracted to the telephone. There is something mysterious about this object that rings from time to time, that stops ringing when part of it is picked up, that you can talk into, and that has voices coming out of it. Many children learn their phone numbers soon after they learn the day and month of their birth.

Teaching young children how to make telephone calls may have some disadvantages but the benefits of learning this skill far outweigh the risks. First, you can ask your child to dial for you when you are making a call. It is a great way for

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children to learn to read numbers, if they are interested in doing so. Then you can teach them to dial their own number and memorize the digits. Your child may make a few "unauthorized" phone calls, but it's reassuring to know that he or she can call you or ask someone to call you in case of need.

If you have older children, you can show them the pattern in domestic telephone numbers: a one-digit access number (1) for long distance calls, a three-digit area code, a three-digit local prefix and a four-digit number that is just yours. (Why are there no local phone numbers that begin with a 1 or a O?) In the first few pages of your telephone book you will find useful information about area codes and local prefixes. The Denver Metro Area phone book has a prefix location guide. If you want to know where the owner of a certain phone number lives, look it up in that guide. Area codes are also listed — on a United States map (which includes time zones) as well as in two tables, one arranged by state, the other in numerical order. You could speculate about all sorts of things by studying the **area codes**.

For instance:

– How many states have only one area code? (Colorado was one of them, until March of 1988.) How many states have more than one area code? Which state has the largest number of area codes?

– Is the size of the state related to the number of area codes?

– Can you tell by looking at the map with area codes where there is the greatest concentration of people?

– California, has the following area codes: 209, 213, 408, 415, 619, 714, 805, 818, and 916. Why do you think the area codes aren't consecutive numbers? Is this true for other states also?

– Look at the table which lists area codes by state: Do you think there is a reason for the way these numbers were assigned to the states? AT&T might be able to help answer this question.

– Look at the table that lists the area codes in numerical order: Can you discover any pattern? Did you realize that all area codes have either a 1 or a 0 as their middle number?

– How many different area codes could there be before the phone companies run out of three-digit numbers?

There are also some **games** you can play with phone numbers. Make a list of the phone numbers of friends and family and ask your kids to estimate which number will have the highest value when all the digits are added together. After estimating, let your children do the addition to see how close they came.

Are the last four digits of your phone number a palindrome? A **palindrome** is a number that reads the same forward and backward. 6336 is a

palindrome, so are 303, 212, 37573 and so on. If your phone number is not a palindrome, you may be able to transform it into one. See "Palindromes" in the ***Appendix*** for directions on how to do this. It's fun and a good activity for practicing addition.

You can ask your children to rearrange all the digits of your phone number so as to make the largest or the smallest possible number. Encourage your kids to rearrange the digits in different ways. How many ways can they come up with?

How do you memorize phone numbers? Most people look for some sort of **pattern** that will make it easier to remember. For many years the last four digits of my work number were 8-4-2-1. That was easy: each successive number had half the value of the previous one. Go on a phone number pattern search with your kids and see what you and they discover.

PHONE BOOKS

Classifying:

Your children could browse through the listings in the white pages, creating categories of names. For instance: look for names of occupations (taylor, barber), names of animals (bear, wolf), names of colors (brown, black), names related to the weather (storm, snow), or to nature (stone, moon), and so on. This may not appear to be a mathematical activity but remember that classification, that is, making categories, requires logical thinking which is a very important aspect of mathematics.

Problem Solving:

The yellow pages of your phone book can give you a lot of information about your community. For example: how many doctors, lawyers, or architects are there? How many movie theatres, grocery stores, restaurants, real estate agencies are there in town? First you could have some interesting discussions with your kids about what's available in your community and why. Then your children could figure out the ratios: how many doctors per person, how many real estate offices? Why are there so few doctors and so many realtors? These are good problem solving questions which also lend themselves to computations of ratios and percentages. (See ***Examples of Computations***.)

Estimating:

How would you estimate the number of residential listings in the white pages of your phone book? At first this may seem like a difficult task but there are many different ways to approach such a problem. Here is one way you can try:

1. Count the number of columns on one page and roughly divide one column into quarters. (The phone book I used had four columns on each page.)

2. Count the number of names in one quarter column and round off the number you count to make multiplication easy. (I rounded 27 to 25.) You are allowed, even encouraged, to do this when you are estimating.

3. Multiply the number of names you counted in one-quarter column by four to get an estimate of the numbers of names in the whole column. (25x4=100) Now multiply 100 by 4 — the number of columns on each page — and you find out that there are about 400 listings on the page.

4. Multiply this number by the number of white pages with residential listings in your phone book. The phone book I used had 135 pages. 400 x 135= 54,000 — it's ok to do this multiplication with a calculator. Here is the answer to the original question: how many residential listings are there in the white pages of your phone book? Approximately 54,000.

This way of estimating is really quite easy: you use some system to reduce a very large unknown number to a small number that you can count. Then you do a series of multiplications to get to the unknown large number. Once you've caught on to the idea it's a lot of fun and with the aid of a calculator, children can gain confidence in dealing with large numbers while practicing multiplication and problem solving skills.

Estimation is sometimes defined as educated guessing. You are not trying to get an exact answer, but a "ballpark" figure, an approximation of the exact answer. Many mathematics educators believe that estimation should be practiced at school all the time, not only because we use it so frequently but also because it helps children to judge whether their computations are correct. If children learn their addition and multiplication facts (such as 3 + 4 =7 and 7 x 8 =56), but do not have a sense of what a reasonable answer to a problem would look like, the rote memorization of such facts will be of little value. This is especially important when using calculators. Children who know how to estimate answers to problems are much more likely to spot calculator errors made by using the wrong buttons.

AWAY FROM HOME

8. GROCERY SHOPPING

Some people like to go shopping, others hate it. It depends, of course, on what you are going to buy. Clothes for yourself or your kids? A labor-saving appliance or some seedlings for your garden? Or the weekly groceries — once again. The following activities have to do with supermarket shopping which becomes a chore for many of us but which we all have to do if we want to have food in our homes. Often we have to do it with one or more of our children. If you are looking for readily available opportunities to practice mathematical thinking, the supermarket is an excellent place. Almost the whole elementary school math curriculum could be taught in a supermarket.

COUNTING

Let's start with counting — a good activity for younger children. If your kids like to count, you can keep them happily busy while you shop. They could count the number of people in the aisle in which you are shopping; the number of children in the store; the number of little kids riding in shopping carts. When you are waiting to check out, your children could count the number of people in the different lines. You might let them select the shortest line. Older children could take an average of all people waiting in line. Take your calculator along!

There are many things in the store that can be counted: how many different kinds of apples, different brands of yogurt, different boxes of cereal? How many aisles? How many checkout lines? How many are open? This would be a good time to teach your child the tally method of recording numbers. (See illustration on page 30.) If younger children are counting orally, write down the results of their surveys: you may think of more ways to use these figures later and you will be showing your child that you value his or her effort.

WEIGHT

Lots of things in the supermarket have to be weighed at the check out and most packages and cans have their net weight printed on the label. Talk about the weight of the things you buy. Let your children feel the weight of a bag of potatoes or a package of potato chips before they place it in the shopping cart. This will sharpen their sense of what is "heavy" and what is "light". Gradually they will be able to translate a feeling for weight into a knowledge of pounds and ounces.

Allow your child to push the shopping cart and comment on its weight as you are filling it up. Is it harder to push now? Why? How would it feel if it were filled up with toilet paper or cereal boxes? What if we bought only potatoes or oranges

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today? With just a few casual remarks like this, you can raise your children's awareness of weight.

Find two packages that weigh the same — for instance five pounds of sugar and five pounds of onions, or one pound of flour and one pound of mushrooms. Do they feel the same? Are the packages the same size? In the produce department, there are scales on which you can weigh the produce you are buying. Let your children help you fill the bags and let them estimate when you have one pound of tomatoes, five pounds of apples or two pounds of bananas. Let your children pick up a cantaloupe, an acorn squash, a large head of cabbage or, if they are strong enough, a watermelon, and ask them to estimate how much each one weighs. At first they probably will just be guessing. With practice and experience, however, their guesses will come closer to the actual weight.

ESTIMATION

Since the price of produce is often determined by its weight, children could estimate how much various items will cost after they put them on the scale: how much will one-and-a half pounds of cherries cost at 89 cents a pound? Make it very clear that you are expecting only approximate answers and encourage rounding off. For instance you could change 89 to 90. At 90 cents a pound the cherries would cost 90 cents plus 45 cents or about $1.40.

You can also estimate how much all your groceries will cost. That may be too hard for your children but if you have some strategies to figure out what you will have to pay for your shopping cart full of groceries, such as rounding off all prices to the nearest dollar, share these with your kids. It's useful for them to watch you go through the steps of estimating and to realize that there is a way to do this.

If your children are interested in your estimations, encourage them to estimate the cost of all your groceries on the next trip. It doesn't matter if they are way off — estimating takes practice — but doing it will still help to develop a general sense of what numbers are in the ballpark. Occasionally, your children could keep track of your purchases with a calculator. This would help them to see how much money is spent on various shopping expeditions and might improve their future estimates.

CLASSIFICATION

Have you ever had the experience of shopping in an unfamiliar supermarket? You spend a great deal of time looking for things because you don't know how the store is organized. Where are the baked goods, the paper plates, the canned juices? Although the overall organization varies from store to store, there are certain large groups of food and non-food items which are always stored together. Knowing what foods have to be refrigerated or kept in a freezer, what foods always go together like milk, butter, and eggs, will help you find your way around more easily. If you are shopping for paper napkins, you can ask your child what other items might be found on the same shelf. Explain the store's organization to your children, it is a good example of how classification is used in real life.

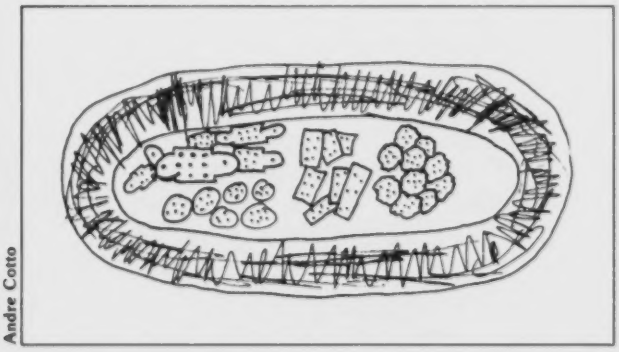
DIVISION

If you are looking for activities involving division, ask your child to figure out the cost of a single item for products that are usually sold in multiples: one lightbulb instead of two, one can of soda instead of six, one stick of butter instead of four, and so on. Somewhat more complicated computations with division are made when you want to compare prices of different sizes of cans of, say, tuna fish or tomato juice. Most large

markets have unit prices marked on the shelves, but there are many occasions when it's useful to know how to figure these. It's easy with a calculator once you understand what to do. Most often the unit for which prices are quoted is one ounce. If a 5 1/2 ounce can of tuna costs 89 cents, the cost of one ounce will be 16.2 cents. (Divide 89 by 5.5 and round off the answer which is 16.18.) If a 12 1/2 ounce can of tuna costs $1.79, the cost of one ounce will be 14.3 cents. (179 divided by 12.5) You are saving 1.9 cents an ounce or 23.8 cents buying the large size can. This kind of calculation is good for problem solving as well as for computational practice. Children may also enjoy doing these divisions with a calculator to see if they get the same number as the one on the unit price marker.

GEOMETRY

You can find many examples of geometry in the supermarket if you go looking for shapes. Cylindrical cans are sometimes stacked in pyramids; circular pizzas, tortillas, and pies are stacked in the freezer, spherical oranges are piled high in produce bins. Crackers come in many different shapes — square, rectangular, round, elliptical, triangular and hexagonal (six-sided). Ask



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your child to hand you that box of **triangular** crackers from the shelves or that **round** pizza in the freezer. Look at the shapes in the various departments: which is the most common? Make shapes the theme for one shopping day — you'll be surprised how many you will see.

TIME

On another day you might try to concentrate on time. How long will it take to get our groceries today? How long will it take us to check out? How do you pick your line — by the number of people waiting, by the speed of the checker or by the maximum number of items allowed? Does the "express" line always move faster than a regular line? Can you estimate how long it will take you to get checked out by the contents of the shopping carts of those waiting in line ahead of you? A stop watch could take the tedium out of waiting in line: it may even interest your kids more than the candy and gum machines.

When you get home, your children can help you unpack, check off items purchased on the sales slip, sort them according to your storage categories, and put them away where they belong.

9. THE POST OFFICE

NUMBERS

What do you do when you go to the post office? You buy stamps, you have packages weighed, you check insurance rates or zip codes, you ask how long it will take a letter to get to its destination — everything has to do with numbers. Just think of the address on one envelope: there is a number for the house in which you live or for your mailbox; if you live in a large city building or a condominium there may be another number for your apartment; your street may be known by a number; the city you live in, or a specific area of your city, is identified by a number — your zip code; the stamps on the envelope have numbers on them; and, though you may not know this, there are size standards for domestic mail, spelled out in great detail in a booklet available at your local post office. (Ask for ***Postage Rates, Fees and Information***.)

SORTING

Today most mail is sorted by machines but in smaller post offices, the incoming mail is still hand-sorted. Have you ever thought about what happens to a letter after it is dropped into a mail box or picked up at your house by your mail carrier? At the post office, outgoing mail is sorted according to its destination — zip codes serve that purpose. When your letter or package arrives in another city or town, together with mail from all over the country, it has to be sorted again so that it will get to the post office closest to your home. There the mail is sorted once more, this time according to the routes of the different mail carriers. When the mail carriers come to the post office to pick up their mail, they sort it according

to street addresses before they start out on their daily deliveries. Think of the huge number of letters mailed every day and the number of times each letter has to be sorted and handled before it finally is delivered to its addressee. It's a small miracle that relatively few pieces of mail get lost.

Sorting is an important mathematical activity because it involves logical thinking. (See section on sorting in ***Laundry*** chapter.) If you receive a lot of mail, let your children sort it into various categories: letters, postcards, bills, magazines, advertisements, and so on.

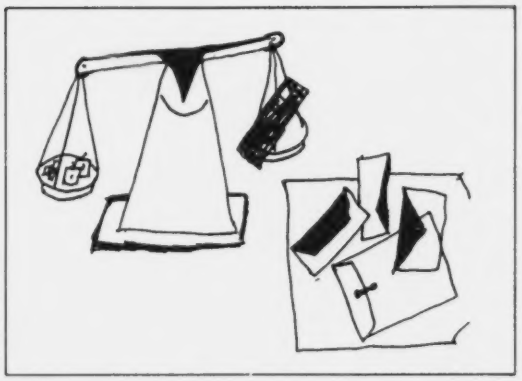
AVERAGES AND PERCENTAGES

Older children could do some interesting computations with your mail. After your mail is sorted, they could figure out, for instance, what percentage of your mail is unsolicited mail — appeals for contributions, catalogs, free introductory offers, and so on, what percentage is bills, what percentage is mail you really appreciate and what percentage goes straight into the waste basket. Your kids, if they are interested, could keep a daily record and calculate weekly and monthly averages. If you have a scale, they could also weigh the incoming mail, and again sort it into various categories. Which category weighs the most, which the least? How much mail do you throw out? Let your kids figure this out both by number of pieces and by weight.

Learning to take averages and percentages is more fun if you can make up your own questions from the family mail than if you practice it only with text book problems that may not interest you.

WEIGHT

If your children weigh your mail and calculate a daily average, you might ask them to think of how much a mail carrier's bag weighs, and how often it must be refilled before all the day's mail gets delivered. If yours is a fairly typical household on the block, you could estimate the weight of all the mail delivered by your carrier. Ask your mail carrier how much the weight of mail varies from day to day and whether there is a maximum amount that carriers are expected to deliver. Perhaps the carrier would let your child lift his or her bag? Feeling weight with their bodies is the best way for a youngster to learn the meaning of the numbers attached to pounds or kilograms.



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AREA AND VOLUME

Birthdays and holidays are a good time to develop your children's sense of area and volume. If you are mailing presents, what size cartons will you need? How much gift wrap do you need for each present you are mailing? Let your children try different ways of fitting wrapped presents into mailing cartons, and let them estimate the amount of gift wrap needed for a variety of large and small boxes. Gift wrapping of boxes is an excellent activity for learning to understand surface area — in this case the area of all six faces (sides) of a box. It takes time and practice to get good at knowing just how much paper will be needed. Some store clerks can estimate to an inch how much gift wrap to tear off their rolls of paper.

CARRIERS' ROUTES (DISTANCE)

Have you ever wondered how many miles or how many blocks a city mail carrier walks each day? Are all the routes the same? Is a hilly route shorter than a flat one? You might ask your mail carrier if all the routes are approximately the same length and whether allowances are made for hilly terrain or for the extra steps that have to be walked and stairs that have to be climbed to get from the street to front door mail boxes.

In rural areas, the carriers drive their delivery routes. How is a route decided in the country? By square area? By number of houses? By number of miles driven? or by the amount of time it takes to cover the terrain? If you live in the country, see if your kids can find out.

JOBS FOR KIDS

Might your children be interested in taking care of your postal needs? You can tell them what stamps to buy or order (the post office will pro-

vide you with special mail-in forms), ask them to weigh your letters and small packages to figure out the postage from charts you can get at the post office, and to put all the necessary stamps on your outgoing mail. Purchasing a postal scale would provide good motivation for this job. Learning how to read different kinds of scales is also a useful and important skill.

If you mail a lot of first class letters, try to buy a variety of different stamps and then encourage your children to make as many combinations as possible to get to the current twenty-five cent postage. Ask them to write down their combinations. How many different ways did they find? Around the first of the month, when you may be mailing out a whole bunch of bills, or on special holidays, when you may be sending a lot of cards, let your youngsters count your letters and figure out the total postage needed.

The postal booklet mentioned earlier in this chapter has some rules about standard size, thickness, and length to height ratios of envelopes. For instance: a letter must be at least 3 1/2 inches high and 5 inches long; a letter will be considered non-standard mail, subject to a surcharge, if any of the following dimensions are exceeded: length- 11 1/2 inches; height- 6 1/8 inches; thickness- 1/4 inch. There is also an "aspect ratio" — length of envelope divided by height — which must fall between 1.3 and 2.5 inclusive. Your children could be put in charge of checking the sizes of your envelopes to make sure they do not require a non-standard surcharge.

TRIP TO THE POST OFFICE

If your children are weighing your mail on a small postal scale, they would probably be interested in seeing the variety of large scales used at the post office. I am sure your children would enjoy a trip to your local post office to see what's going on "behind the scenes." See if you can arrange it at a relatively quiet time of day.

10. RESTAURANTS

Many families take an occasional trip to a restaurant. This is a time to relax, you may be saying to yourself, not to think about mathematics. True, but maybe your children might enjoy some problem solving activities while they wait for their food. Below are some suggestions for restaurant mathematics.

PAYING THE BILL

Paying the bill is the first mathematical activity that comes to mind when thinking about restaurants. Take your pocket calculator along when you go to eat out with your family. After ordering your meal, your children can total up your bill from the menu. If older children are studying percentages in school, they can figure out the tax for the bill and then calculate how large a tip you should leave. When you get your bill, the children can check their calculations against the numbers on the bill.

All this is easy to do with a calculator but be sure the children understand what they are doing. Just pressing the percentage button on the calculator before knowing what percentages are is not useful, because children have no way of telling whether the number in the calculator window makes any sense.

Figuring a fifteen percent tip is not a difficult computation, even without a calculator, once you understand percentages. First figure out how much ten percent would be. If the bill for the meal is $15.78, ten percent of that amount is $1.578 or, rounded off $1.60 (10% is 1/10th. You divide 15.78 by 10 by moving the decimal point one digit to the left.) Then you take half of ten percent ($1.60) to get five percent (80 cents.) Add 80 cents to $1.60, and your fifteen percent tip comes to $2.40. In some communities you can use the tax that is added to your restaurant bill to figure out a fifteen percent tip. If the tax is approximately five percent, just multiply that amount by three. If the tax is seven or eight percent, double the amount of the tax.

MENUS

If there are restaurants in your neighborhood which deliver pizzas and other food, try to collect their menus. When you place an order, let your children figure out how much money you need to have ready when the food is delivered. You could also collect menus from other restaurants — many of them will let you have one if you ask for it, especially if you explain why you want it.

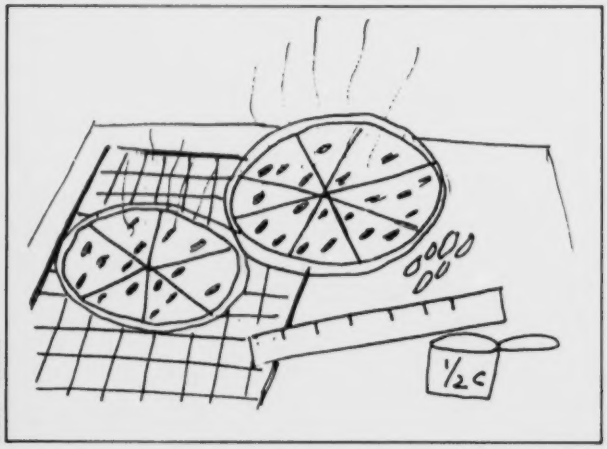
With a collection of menus, children can plan imaginary meals and see how much they would cost; you could also give your kids some menu problems such as planning a meal for two for under ten dollars or a meal for five for no more than thirty dollars. Using real menus and figuring out the total of various meals is not only good practice in addition. It also will give children a sense of what it costs to eat out.

PIZZA MATHEMATICS

Many pizza parlors display cardboard samples with prices of the different size pizzas sold. Sizes are given in diameters — the measurement across the center of the circles. Knowing the **area** measurement of each pizza size will help you to make the most economical selection. You can get good estimates of area without any computations. For instance:

If you have a ten-inch and a fourteen-inch pepperoni pizza, count the number of pepperoni on each. How many more are there on the larger pizza? Did you expect this difference?

Draw ten-and fourteen-inch circles on large pieces of paper. Cover the circles with a layer of beans. Then pour the beans into a measuring cup, one circle at a time, and compare the amounts.



Kate Andrias

Draw ten- and fourteen-inch circles on graph paper or use blank paper and mark it off with one-inch squares. Count the number of squares in each circle and compare the results. (If more than half of a square is in the circle, count it as a whole square; if less than half is within, don't count it.)

To compute the area of different size circles, you use the following formula: $r^2 \times 3.14$.

"r" is the radius of the circle. (The radius is half the diameter.)

r^2 means radius squared or radius multiplied by itself. (For example: seven squared is 7 x 7)

3.14 is the approximate value of pi written π. All you need to remember to find the area of the pizza is to multiply the radius of the pizza by itself and then by 3.14.

Here is how you calculate the area of the ten inch and the fourteen inch pizza:

10" Pizza 5 x 5 x 3.14 = 78.5 square inches.

14" Pizza 7 x 7 x 3.14 = 153.86 square inches (154 sq. in. rounded off.)

Notice that the area of the 14" pizza is almost twice as large as the area of the 10" pizza. Is that what you found out when you counted pepperoni and squares and measured beans?

Knowing the area measurements of pizzas is useful because you can now compare prices and see which size pizza gives you the most for your money. The price for a 10" pizza in my neighborhood pizza parlor was $4.49. A 14" pizza cost $6.14. If you buy one 14" pizza at $6.14 rather than two 10" pizzas at $4.49 each, you'll be saving $2.84 for almost the identical amount of pizza. You may want to find the area measurements of all the sizes sold in your favorite pizza parlor. Compare the area with the prices and you'll always be able to pick the best buy.

Some of the above calculations as well as a few others in this booklet may well be too difficult for your children but they are included because **you** might enjoy doing them. Remember that your child can become acquainted with a concept and get a general idea of what calculations are about without being able to do all the computations. Do the computations for your own enlightenment and let your children watch you. Then try to explain the results to them on a simple level. It is important that your children see that you, too, are interested in math and enjoy solving problems.

Pizzas are also useful for practicing geometry. You can divide pizzas in many different ways. Show your children how to make four, eight or sixteen slices by repeated halving, and how to make six, or twelve sections by starting out with thirds and then cutting these in half again. Do you ever make five or seven slices? If you don't want to get involved with angle measurements, you'll have to use your eye. The best way is to go around the circumference of the pizza, the outside rim of the circle, and divide that up into even sections of five or seven before you start cutting.

PLAYING WITH NUMBERS WHILE WAITING FOR YOUR FOOD

While waiting in line in a fast food shop, you can make up some interesting estimation problems. For instance: how many french fries does the shop serve in a day? I won't tell you how to solve this problem, but I will tell you how a friend

of mine, who loves to play around with numbers, answered a similar though somewhat more difficult question that he put to himself: What is the annual gross intake of my local supermarket? This is how he proceeded:

He spent about fifteen minutes observing people passing through the check-out lanes. First he estimated the average amount of people's purchases ($25). Then he counted the number of check-out lanes and asked the manager how many were open and for how many hours. He estimated that on the average, eight lanes were open and that about ten people passed through one lane in one hour. He figured out that the store was open an average of sixteen hours a day. Then he began to multiply: average purchase price ($25) multiplied by number of lanes (8), by the number of people per lane in one hour (10), by the average number of hours the store was open (16) by the number of days the store was open in the year (365): $25 x 8 x 10 x 16 x 365 = $11,680,000 which he rounded off to ten million. He checked this figure with the store manager who told him that it was surprisingly close to the previous year's annual gross income. For a relatively quick estimate, that is an excellent approximation.

Can you transfer my friend's method to your local MacDonald's french fries' consumption? Remember that you want a "ballpark" figure and try to approach this question in the spirit of playful mathematical exploration.

There are simpler things to figure out while waiting for your food or for your bill in a restaurant. Look around the room: how many booths are there? How many tables? Are the tables arranged in a pattern so that you could easily count them? Look to see how many people there are at each booth and table. Can you figure roughly how many people there are in the restaurant? How many people would there be if every seat were taken? Can you observe (or ask) how many tables each waiter or waitress is responsible for? What is the ratio of waitress to guests? How many people are served per night by each waitress? How much money might she make in tips? You will find out that once you raise questions like this, more and more questions will come to you. This kind of playing around with numbers does not appeal to everyone but if it should catch your and your children's fancy, it could become an enjoyable way to engage in mathematical thinking.

TRIP TO A RESTAURANT KITCHEN

A look at a restaurant kitchen gives children an opportunity to see the familiar shapes of pots and pans in much larger sizes. The mathematical concept of similarity — objects that have the same shape but different sizes — can be well observed here. I have always been fascinated by very small and very large things that serve the same purpose as the middle-sized objects with which we are most familiar. For instance: doll-sized pots and pans, our own family-sized pots and pans and the very large commercial pots and pans used in restaurant and hotel kitchens. Talk to the restaurant manager and ask if you could bring your children to see the kitchen at an hour that would be convenient. Children of all ages, as well as adults, would find such a visit interesting and instructive.

Many people think of "educational" trips as going to museums or historical sites. Yet almost everything around us has some educational value if you are willing to spend a little extra time. In this book, I suggested trips to a post office, a restaurant kitchen and a lumberyard. There are, of course, many other places that you can visit with your children to watch different people at work.

Elizabeth Kellogg

A trip to the Colorado Brick Factory

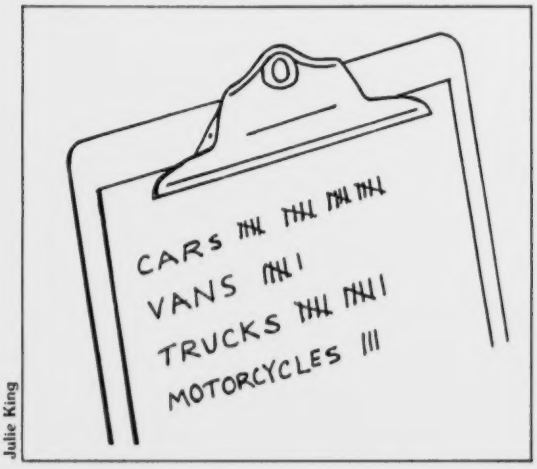
11. CARS - TRAVEL

Many children spend a good deal of time riding in the family car and some of them become fascinated with different aspects of cars and driving. The instrument panel with all the gauges is a special favorite.

Take advantage of your children's interest and of the time they spend in the car to stimulate their mathematical thinking.

AT THE GAS STATION

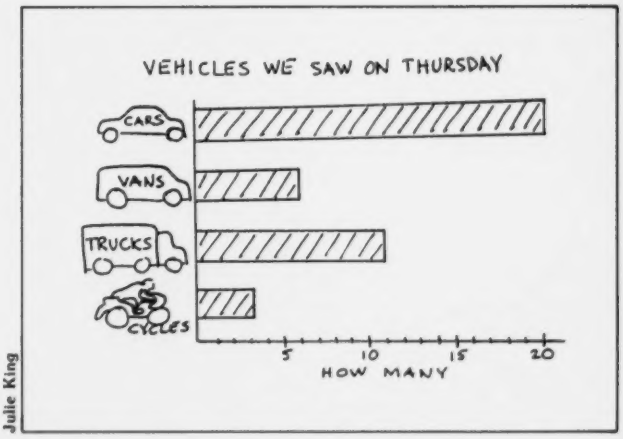
If you go to self-service stations, your child can keep track of the number of gallons of gas going into your tank as well as of the amount of money the gas will cost. If you want ten gallons, or ten dollars' worth of gas, ask your child to watch the gauges and tell you when to stop pumping gas. Children can also estimate how much ten or fifteen gallons of gas will cost at various prices. If gasoline costs 86 cents a gallon, will ten gallons cost more or less than ten dollars? What if it costs 94 cents or $1.05 a gallon? These are good estimating problems for children who have some experience with multiplication. Do you know how many gallons your tank holds? Can your children figure out how many gallons you can add if the tank is half full? One quarter full? Can they estimate how much money you will need in each case?



Julie King

TRAFFIC COUNT

If you live on a busy street, you can have your youngsters take a traffic count. Teach your children how to keep track of the cars by tallying. At first they may just count all the vehicles that pass by during a certain period of time. Later, each child could keep track of a different type of vehicle — cars, pick-ups, larger trucks, and so on. Let your kids make up their own categories. You might suggest that they count the vehicles at different times of the day, and on different days of the week. An interesting story might emerge from their surveys. If they have done some graphing in school, they could put all the information they collected onto a simple bar graph. If anyone's birthday is coming up, a clicking counter would make a fine gift.



Julie King

HOW MANY MILES PER GALLON?

When did you last figure out how many miles per gallon you get in your car? Is it the same whether you drive mostly around town, take a trip to the mountains, or drive on the interstate? Do you use more gas going 65 mph than if you go a steady 50 or 55 mph? Let your child keep track of your mileage and the amount of gas you use over a period of time.

Can you help your children understand why start-and-stop city driving, mountain roads or higher speeds will increase your gas consumption? You might compare this to the energy they would expend running very fast, running uphill, or running and constantly stopping and starting again. They can also try to push a heavy object: is it easier to get it started or to keep it going once it is started?

A CAR BOOKLET

Some children might enjoy keeping track of car related facts and figures, at home and on trips away from home. Give your child a special little car notebook in which he or she can record some of the following:

At home: – mileage and date when you change your oil;
– mileage and date when you rotate your tires;
– date on which you last checked your anti-freeze as well as temperature to which it was safe;
– correct tire pressure, front and rear wheels;
– any other things that get checked at regular intervals;

On trips: – number of miles driven — recorded daily, when you fill up your tank, between cities, or any other way that makes sense to your kids;
– amount of gas used — record whenever you buy gas;
– cost of gas: prices will vary so children should record price per gallon as well as total cost each time you fill up your tank.

At the end of the trip, older kids could make the following calculations:
– total number of miles driven;
– total cost of gas;
– average cost of one gallon of gas; (total $ paid for gas divided by total gallons purchased)
– average number of miles per gallon of gas; (total number of miles driven divided by total number of gallons purchased)

Out-of-State License Plates

Recording out-of-state license plates is a popular game. While riding in the car, at rest stops, in parking lots in picnic areas, children often like to look for out-of-state license plates. Let them keep a tally of the number of out-of-state license plates seen. They can also copy down the letters and/or numbers on the plate plus any other message.

Road signs give a wealth of information. For example:

Shapes of road signs:
- How many different shapes are used?
- For what purposes are different shapes used?

Speed limit signs:
- How do you know whether a posted speed limit is suggested or required?
- Can you explain why speed limits change?

Andre Cotto

Steepness of mountain roads expressed in percent grade:
- Why are trucks asked to shift into low gear when driving downhill on a steep road?

Weight or height limits for trucks:
- Where do you see these signs and why?

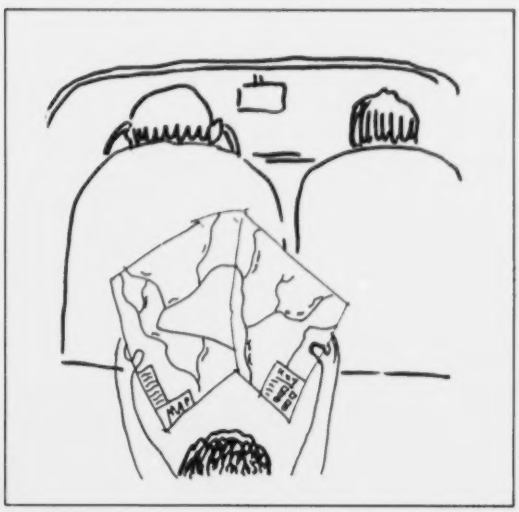
Discuss with your children why speed limits change, why heavy trucks can't go over certain bridges, or why drivers are alerted to steep grades. It is important for them to learn that numbers play an important role in communicating all kinds of information.

Children could also draw and write in their car book, especially on trips. They could sketch how cloverleaf interchanges are laid out or make pictures of how the shadow of the car looks at different times of the day and how it changes when you change the direction in which you drive. Any kind of drawing, however, could go into their booklet, as well as thoughts, feelings or ideas that they may have on a trip.

ROAD MAPS AND SYMBOLS

Road maps contain a lot of useful mathematical information. Although no longer available free of charge at gas stations, you can get one free Colorado road map at your local Chamber of Commerce. The cost of other maps at gas stations is well worth the investment. Becoming familiar with maps and learning to read them is an important and useful skill.

To help children understand that maps can be drawn to different scales, you might get several different maps which include the State of Colorado. For instance, a Colorado map where one inch equals 18 miles, a map of the Western States where one inch equals 49 miles, and a United



Andre Cotto

States map where one inch equals 90 miles. The idea that a map can be made to any scale is important but not easy to grasp for young children. Perhaps your kids could try to make a "map" of their room on graph paper. Then you can show them how to reproduce it to a smaller or a larger scale by using graph paper with smaller or larger squares.

Maps have many symbols, all explained in a key or legend at the side or bottom of the map. You can identify state capitals, tell how large a city is, where the parks and recreation areas are located, and what sorts of roads there are: interstate highways, paved state roads, dirt roads, and so on. There are grids with numbers and letters (called coordinates) at the four sides of the map, which divide the map into sections so you can find a particular place. Ask the children to find different cities or towns. This

Andre Cotto

will provide good practice in locating a point on a grid, a skill that will be needed later in school when coordinate graphing is taught.

WHEN ARE WE GOING TO GET THERE?

Time spent sitting in a car can pass very slowly for children. Many families play games to help pass the time. Here are some additional activities to keep your children interested and busy:

Can the children find and follow the numbers of the roads on which you are traveling on a map?

If the children can read maps, let them figure out how many more miles it is to a certain place and how much time it will take at the speed you are driving; or tell them at what time you want to stop for lunch and let them figure out on the map where you will be.

Compare travel time given in the road atlas with your own travel time; figure out traveling time at different speeds; figure out how much less time it would take you to get to a place if you could go "as the crow flies".

Compare different routes as to distance and traveling time and let your children participate in the decision making when you have choices of routes. Scenic vs Interstate, tunnel or mountain pass, detour for a land mark or time spent at a picnic area, and so on. It is important for children to

learn to make decisions and to see what the consequences may be. For instance, they may have opted for the mountain pass without realizing how much longer that will take.

Estimate how far it is to some object in the distance — a grain elevator, a water tower, an overpass, a rock formation. Let everyone in the car make a guess and then watch the odometer till you get there. Initial guesses may be way off but improvement usually comes quite soon.

SPEED

Could you calculate your driving speed if you did not have a speedometer? When the speedometer on your car reads fifty, do your children know what the fifty stands for? A teacher told me that some of her high school students had a difficult time with text book word problems about speed, time and distance. When she asked them how long it would take them to drive to Denver (a distance of 100 miles) at 50 miles per hour, they answered without hesitating "two hours." When she gave them the same problem worded in text book style — "The distance between point A and point B is 100 miles. Mr. Jones leaves point A at 10 a.m. and drives at a steady speed of 50 miles per hour. At what time will he get to point B?" — the students said they couldn't do the problem. Formulas like distance divided by speed equals time (in this case 100/50 = 2) are even more confusing to students when they cannot connect them with their experiences.

The concept of **miles per hour** is not hard to understand when you are actually riding in the car where you can keep track of the time and watch both odometer and speedometer. Lots of talking with children about how fast you are driving, how long it will take to drive sixty miles, or how far you will be driving in this time will help to familiarize them with the idea that there are measurements that depend on two variables. If **miles per hour** is well understood, children may have less difficulty with other similar measurements like **revolutions per minute** or **dollars per capita** which they will learn in later school years.

MISCELLANEOUS

12. MONEY MANAGEMENT

A recent TV program showed a group of upper elementary children being interviewed about money. The children were asked whether they received an allowance, whether they had jobs to do at home, where they kept their savings (mostly in piggy banks or jars) and, finally, where they thought that money came from. In answer to this last question, most of the kids said that money came from banks. A smaller group thought it came from their parents, quite a few children said they did not know, and only a small percentage connected money with working. Clearly there is a need to help children understand early in life that most people must work to earn money if they want to have it to spend.

ALLOWANCES

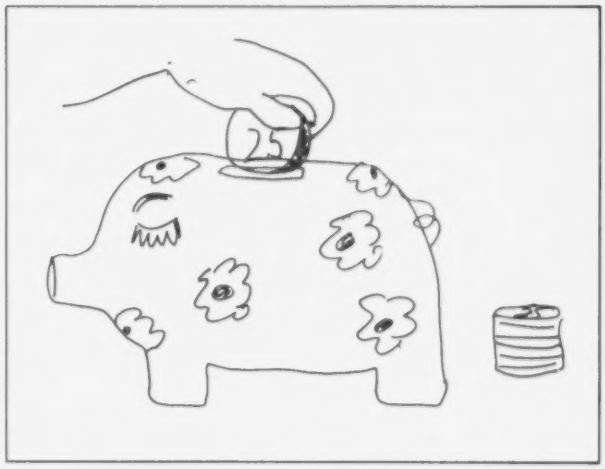
How can parents help children understand money? Giving them an allowance is one way. By getting a fixed amount of money to spend, children can learn to make choices. If they buy all the little things they want — gum, comics, trinkets — they will never have enough money for bigger items. Talk to your kids about the choices you have to make: a new dress or a new pair of shoes; a family camping trip or a new TV.

Some parents feel that an allowance with no strings attached is the best way for kids to learn how to spend money sensibly while others make it clear that they expect their children to pay for certain things, like school supplies or movies, out of their allowance money. Other parents feel that children should learn early that you don't get money without working for it. They might give children some chores at home to earn their allowance. These are obviously individual decisions made by each family according to their needs and their beliefs.

As your children grow older, you can increase their allowance and make them responsible for more of the purchases that you have previously made for them. A friend told me that when she was in high school, she was expected to buy all her clothes out of her allowance. She didn't like it at the time but she learned to become a careful shopper.

SAVING MONEY

How can you help children to learn to save money for bigger purchases? Again, talk about the choices that everyone has to make but also stress that small amounts regularly saved do accumulate over time. Figure out with them how much money they would have to save for something they want to buy. How much do they have already? How much more do they need? How many weeks would it take them if they saved, say $1.50 a week? If you can afford it, you might offer to match your children's savings so they would have to save only half the necessary amount. It is easier for many children to save money if the amount they need or want does not seem out of reach.



Eve Andrias

Opening a savings account for a child will often give a big boost to his or her desire to save money. I recently read that Denver has a savings bank just for children. Having your own bank account, going to the bank to deposit money, watching your money grow through deposits and interest — these are all ways to increase your child's awareness of what money is and how it works.

EARNING MONEY

Most children love to earn money, partly for the sake of having it to spend, and partly because it is a grown up thing to do. Older children can babysit, wash cars, shovel snow, cut grass, help with yard work and household chores, sell lemonade and run errands. Discuss with your kids how much to charge if they work for other

people; help them figure out how much they could earn in one day and how many days they would have to work to buy a desired item. How much could they earn in one summer?

WRITING CHECKS

If you let your child watch you write checks and deposit slips, and explain in general terms how a checking account works, children might be ready in 5th or 6th grade to manage their own account. If you have a checking account, could you give one of your check books to your child and then become his or her banker? (If you don't want to use your checks, you may be able to get some blank ones from your bank.) You might "open their account" with a birthday gift of $25. Whenever they earn some money or are due for an allowance they can enter the amount in their checkbooks rather than taking the cash. When they need to withdraw money, they will write out a check and get the money from you.

BEING A THOUGHTFUL CONSUMER

If you do comparison shopping, save coupons, take advantage of sales and specials, and patronize discount stores, explain to your children how you arrive at decisions about where to shop. Show your children the advertising supplements for food and other sales and encourage them to browse through them. You might give them your shopping list and tell them to pick the best bargains. Ask them to add up the savings for a week's shopping. If they select several different stores for you to get your groceries, help them to understand that other factors, such as time, gas cost, and possible parking fees also have to be considered.

If you clip coupons, you might let your youngsters manage them for you with the understanding that coupons will be used only for items the family really needs. You could offer to pay your kids half the savings for their work. They may want to categorize the coupons, keep track of expiration dates, check coupons against items on your shopping list, and so on. Ask your kids if coupons (and special sales) always save you money and make them aware of the temptation of buying things you don't really need just because they are on sale.

Here is an interesting item pointing up the need for consumer awareness: The January 1988 issue of ***Consumer Reports*** had a story about paper towels. An alert shopper had noticed a change in the number of sheets per roll in a certain brand of paper towels. She mailed the wrapper panels from five rolls which she had bought to the magazine. Here are the paper towel statistics in the order in which she bought them.

100 sheets	11.05 x 11.1 inches
	85 square feet
100 sheets	11 x 11 inches
	84 square feet
92 sheets	11 x 11 inches
	77 square feet
88 sheets	11 x 11 inches
	73 square feet
84 sheets	11 x 11 inches
	70 square feet

The price remained the same. How many shoppers, I wonder, realized that their rolls of paper towels were shrinking? Sixteen fewer sheets on each roll account for a loss of fifteen square feet — that's a 16% drop in number of paper towels per roll and an 18% drop in area. Without a corresponding drop in price, this gradual shrinkage is equal to a 19-21% price increase yet the manufacturer said he had not raised the price.

THE VALUE OF DIFFERENT COINS

In pre-school, children often use play money in their dramatic play. They have watched their parents pay for purchases with dollar bills and coins and they probably have observed that the sales clerk often returns some money. Yet they have little if any understanding of the value of the various bills and coins. By first grade children begin to recognize coins and may be able to tell you their worth but it will take a while longer before they really understand the value of the coins. Often they think that the larger coins are worth more than the smaller ones; a nickel is generally chosen over a dime.

You could make a "coin board" (see next page) to familiarize children with the value of different coins.[1] Collect a number of small items costing less than a dollar and label them with price tags. Give your kids $2.00 worth of change: 50 pennies, 10 nickels, 5 dimes, and 2 quarters. Then ask them to find out how many different ways they could pay for each of the items using the available change. For each way, children can put the coins on a separate row on the board.

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[1]The idea of a coin board is taken, with permission, from ***Family Math*** by Jean Kerr Stenmark, Virginia Thompson and Ruth Cossey. Lawrence Hall of Science, University of California, Berkeley, CA 94720, 1986, pp. 38-39.

COIN BOARD			
25c	10c	5c	1c

Adapted from *FAMILY MATH*

For instance: if you have a colored marker costing 17 cents, you could pay for it in the following ways:

Dimes	Nickels	Pennies
1	1	2
1	-	7
-	3	2
-	2	7
-	1	12
-	-	17

Note that there are six possible ways you could pay for this marker. Ask the children to record how many ways there are for each item they "buy" and point out the interesting patterns that may arise as they use different combinations of coins for their payments.

It's best to start with a small number so that children can see all the possible combinations. The larger the amount, the more possibilities exist. You can also make certain rules: use no more than a certain number of pennies, use as many coins as possible, use the smallest possible number of coins, and so on.

MAKING CHANGE

I once watched a math lesson in a third grade class in which the children were learning to make change. They got very confused because they were asked to count up from the purchase price to the amount which they were supposed to give to the sales clerk. This seemed like adding to the children who thought that they should be subtracting. Although most larger stores have cash registers that calculate the change, there are still places where the clerks make change by counting up. It's important for children to understand the reasoning behind this method.

Let the children "buy" one of the items you collected for the coin board and show them how change is made. If they make a purchase of $1.27 and give you two dollars, this is how they can calculate the change:

$ 1.27 purchase price

+ .03	three pennies	= $1.30
+ .20	two dimes	= $1.50
+ .25	one quarter	= $1.75
+ .25	one quarter	= $2.00
$.73	change	

Store clerks generally make change with the fewest possible coins but after your children get the idea, they may want to practice making change in different ways. It's fun to do and it's excellent practice in addition and subtraction.

MENTAL MATH GAMES

Here are a couple of games you can play with your older kids when you have to wait at places like the post office, the dentist, or the bus station. Tell your children how many coins you have in your pocket or change purse as well as the total amount of money you have and ask them to guess what coins you have. Or tell them just the amount of money you have and let them guess the coins — that's a little more difficult.

The three activities described above — coin board, making change and guessing what coins you have — give children lots of practice with money but beyond that they are excellent problem solving activities. Encourage your kids to make up their own problem solving questions; it's at least as valuable, if not more so, than answering questions others have made up.

13. HOBBIES

People have so many different hobbies that it would be impossible to discuss them all. This chapter gives a sampling of a few popular hobbies: needlework, art and crafts projects, carpentry, and model-making, emphasizing the mathematics of these activities rather than the "how to" aspects which can be found in many good books. Children should see, over and over again, that mathematics is part of their lives outside of school, that it is not confined to arithmetic problems in textbooks or workbooks and that it enters into almost everything people do.

NEEDLEWORK

If you like to sew and enjoy making your own or your children's clothes, do share your knowledge. You do a lot of measuring when you sew. How do you adjust the pattern to fit your own or your child's measurements? If you have material at home, or if you have to buy it, show your child how to read the table on the pattern envelope and figure out if you have enough. Will your pattern fit on the fabric? How do you arrange the pattern pieces so that the largest possible piece of fabric is left over? Children can get a lot of good practice with area measurements if they learn how to fit fabric onto patterns or even learn to make up their own patterns.

Knitting, crocheting, simple weaving and stitchery all create patterns. Even kindergarten children can learn these crafts and can enjoy their creative efforts[1]. In working with these crafts, children have to do a lot of counting and become aware of how numerical patterns translate into different designs.

ARTS AND CRAFTS

If you have no qualms about using food products for art work, you can create wonderful mosaics with a variety of beans. Cut a small piece of heavy cardboard or wood and give your child a collection of different beans — black beans, pinto beans, kidney beans, small lima beans, navy beans, black eyed peas, or whatever you can find in the stores. (Health food stores are particularly good places to buy beans for an art project since you don't have to buy a whole package of each kind.) You might want to show your children pictures of some old mosaics from an art book. Then let them play around to work on a design. When they know what they want to do, give them a small piece of wood, just large enough for their design. Show them how to spread Elmer's glue on a small section of the wood and then have them transfer the beans onto that part of the wood. Since the glue hardens pretty fast, it's best to work one section at a time. After all the glue has completely dried, the beans, if properly placed, will remain firmly attached. Very attractive designs can be made with different colored and shaped beans. In mathematics, completely covering a surface with a regular pattern of shapes is called **tesselating**. Tiled floors and patchwork quilts are examples of tesselations. Here you are tesselating with beans.

Elizabeth Kellogg

A nice art project that also has mathematical content (balancing) is the creation of a mobile. You can attach objects with string or yarn to a wire coat hanger or a branch with an interesting shape. Attach a string to the coathanger hook or to the branch and suspend them from the ceiling or some other high point. (You have to experiment with the branch to find the point at which it will balance.) Cutting out different shapes and suspending them from the mobile while keeping it in balance will give children an intuitive feeling for the area of the various shapes.

CARPENTRY

Children of both sexes love working with real carpentry and woodworking tools. Let them help you measure before you make a cut. Let them see how separate pieces of wood fit together to make a book case or a cabinet and give them lumber scraps to construct their own boats, aircraft,

[1]There is a wonderful book, ***Stichery for Children*** by Jacqueline Enthoven, published in 1968. Unfortunately it is out of print but try to get it from the library — it has excellent directions and many illustrations of children's work.

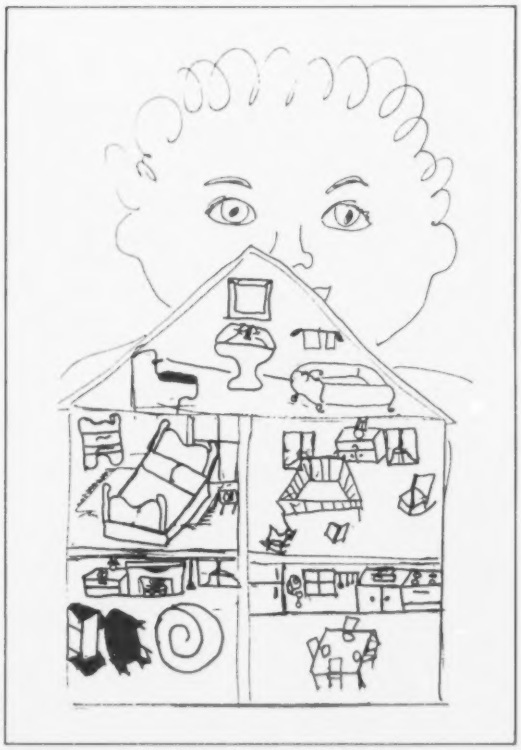
garage for toy cars, or simple doll house furniture.

Take your child along to the lumberyard — a fascinating place where almost everything is sold by measurement. Boards come in various thicknesses, widths and lengths and are sold by the "running foot." Let your youngsters figure out which length board you should buy so you can cut your shelves with a minimum of waste. Let them measure a two-by-four piece of wood to find out how wide and thick it really is. Was it ever two by four inches?

There is much to learn by looking around at the lumberyard. We always tell children that wood comes from trees but some young children find it hard to understand how a round tree trunk is transformed into flat boards. You could slice a zucchini the long way and let them see what happens.

MODEL-MAKING

If your kids are interested in model making, they will gain excellent experience planning out a project or reading instructions for putting together a model. Many people have difficulties understanding instructions on how to assemble an object. The instructions are often poorly written and if you don't have experience translating two-dimensional drawings into three-dimensional objects, you can get very confused. Any help you can give your youngsters here will pay off in later geometry classes as well as in their own lives.



Eve Andrias

Homemade doll houses can be lots of fun to construct. A simple one can be made out of a carton with one side cut out. Let your child wallpaper the inside, make furniture, electrify it, and find or make people who fit into the doll house. Very elaborate doll houses can be purchased or viewed in museums. If you live in Denver or go there for a visit, you can see some of these at the Denver Art Museum. Children will have a greater appreciation of this wonderful collection of old doll houses if they have constructed their own versions.

There are all kinds of unforeseen and mathematically fascinating problems that arise when you try to create a miniature world true to scale. Queen Mary of England ordered a replica to be made of some rooms in Buckingham Palace. Her expert craftsmen made some unexpected discoveries. For instance: the linen table cloth for a tiny table would not hang down over the edges. It stuck out rather stiffly because linen cannot be woven thin enough to hang when it covers a very small area. The water would not flow through the narrow pipes, it hung on the spout in a large drop. This is all related to length, area and volume — the mathematics of scaling. It's a complicated subject not always easy to grasp and rarely taught in schools. Yet just becoming aware of these phenomena is important and can be of great interest to children and adults alike.

Dioramas are also fun to make — the Denver Natural History Museum has large displays of animals in their natural settings. I saw small dioramas in shoeboxes which were final projects in classrooms where children had studied birds and dinosaurs. Each child studied a particular bird or dinosaur. After the children had learned all about the different habitats of their chosen animal, they collected materials to recreate the appropriate environments. In dioramas, as in dollhouses, scale is a key factor.

14. WEATHER

Weather is a universal topic of conversation during all seasons of the year. It is also a great subject for children and offers opportunities for practice in three elementary school mathematics topics: measurement, statistics, and probability.

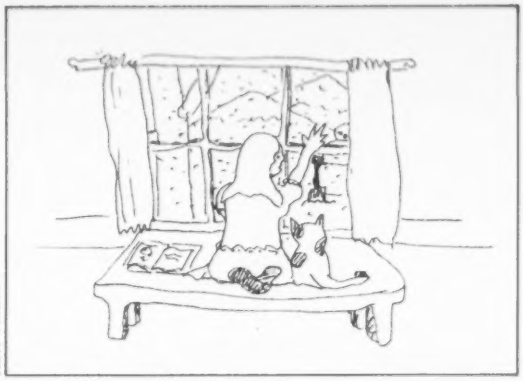
Most of us watch the weather news and are familiar with the following measurements: temperature, including maximum and minimum for the day; wind velocity and direction; barometric pressure; amount of rain and snowfall; and chance (probability) of rain, snow, sunshine or high winds. Your children can take many of these weather measurements at home.

AMOUNT OF RAINFALL

Do you know how to measure the amount of rainfall? You can just put out a coffee can and measure the amount of rain water that has collected in the can with a ruler. If, for example, the water comes up to the two inch mark, two inches of rain have fallen. Most rainfalls, especially in Colorado, measure just a small fraction of an inch, so it's hard to get accurate readings with this method. A rain gauge collects the rain in a wide funnel which narrows down and is placed inside a narrow bottle. It makes a small amount of rain easier to measure because the water comes up higher in the bottle[1].

WATER CONTENT OF SNOW

In the dry climate of the Front Range, we depend on snowfall for our summer water needs. Communities which get their water directly from a mountain reservoir or watershed take regular monthly readings of the amount of snow that has accumulated (the snow pack) as well as of the water content of the snow. We know from experience that the water content of snow can vary a great deal: fluffy powder is light, easy to shovel and great for skiing, but because it contains little water, it's not so great for next summer's water supply. The heavy spring snows, on the other hand, which crush tulips and daffodils and can break off large branches from trees if the leaves are out already, will fill our rivers and reservoirs.



Kate Andrias

A simple way to measure water content is to put snow into a container with straight sides, like a tall can or glass jar, and then let it melt down. You can push the upside down container into the snow or fill it up with a large spoon being careful not to compress it. Measure the height of the container. After the snow is melted, measure the height of the water. If the container is ten inches high and the water is one inch high, the water content of that particular sample of snow is one out of ten, or ten percent. If you have made a rain gauge, you can pour the melted snow water into your rain gauge to measure smaller amounts of water. Be sure, though, that the diameter of your snow container is the same as the diameter of the top of your rain gauge funnel or you won't get a correct reading.

This is not an exact way of measuring water content but it is a good beginning. Initially, young children will be most interested in the dramatic disappearance of the snow as it melts. Very dry snow will leave amazingly little water because it has so much air between the snow crystals. If your children do a lot of experimenting with melting snow, they may notice that the amount of water varies. That would be the time to suggest measuring the water content. Talking with a snow expert at the University of Colorado, I learned that the average water content of snow in Colorado is about six to seven percent, though it can sometimes be as low as two percent or as high as twelve percent. In the Cascade Mountains of Washington, snow can contain as much as twenty percent of water. Children need to have some understanding of percentages and ratios to appreciate such measurements.

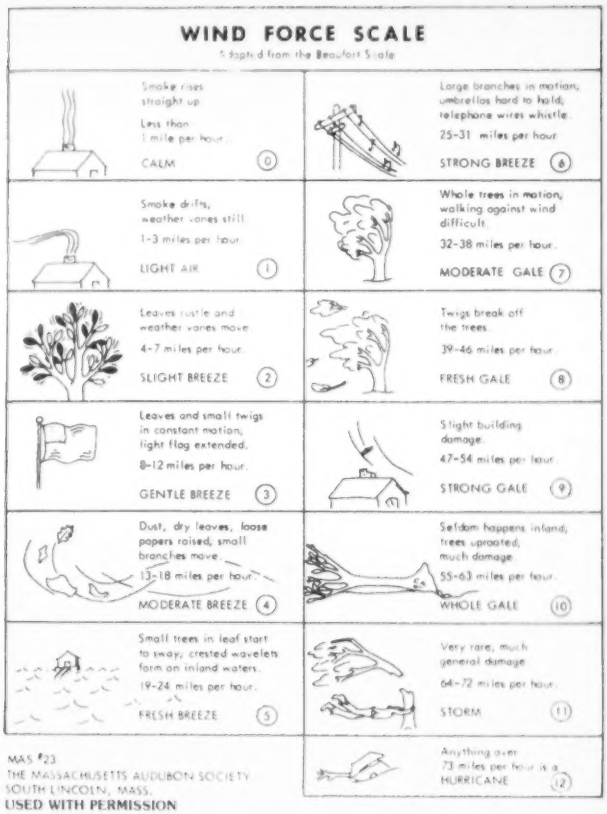
[1]Directions for making a simple rain gauge as well as other weather measuring instruments can be found in ***Everyday Weather and How It Works*** by Herman Schneider, Whittlesey House, 1961. Unfortunately, this excellent and easy to read book is out of print but many children's libraries have copies of it.

WIND SPEED

What about wind speed? Have you ever noticed the wind sock at a small airport? It gives the wind's direction but it also gives a rough indication of wind speed by its position — is it hanging down limply or is it sticking out at a right angle to the pole to which it is attached? Look at the picture chart which gives approximate wind speeds based on the movement of flags, smoke, falling leaves, and so on. Watch flags, smoke, tree branches or laundry on a line on a windy day and estimate the force of the wind. Then listen to your local weather news and compare your estimates to the wind velocities reported. The chart will help you with your guesses.

WIND CHILL FACTORS

On cold days with strong winds, the weather news gives us wind chill factors — temperature figures which tell us what the actual temperature **feels** like at different wind speeds. Look at the wind chill chart: an actual temperature of twenty degrees Fahrenheit will feel like sixteen degrees with a five m.p.h. wind, but if the wind blows at fifteen m.p.h., twenty degrees will feel like minus five degrees. If you've ridden a bike down a steep hill on a cold day, these figures won't surprise you. Have you ever wondered why motorcyclists wear heavy jackets on warm summer days?


WIND FORCE SCALE
Adapted from the Beaufort Scale
Smoke rises straight up.
Less than 1 mile per hour.
CALM 0
Smoke drifts, weather vanes still.
1-3 miles per hour.
LIGHT AIR 1
Leaves rustle and weather vanes move.
4-7 miles per hour.
SLIGHT BREEZE 2
Leaves and small twigs in constant motion; light flag extended.
8-12 miles per hour.
GENTLE BREEZE 3
Dust, dry leaves, loose papers raised; small branches move.
13-18 miles per hour.
MODERATE BREEZE 4
Small trees in leaf start to sway; crested wavelets form on inland waters.
19-24 miles per hour.
FRESH BREEZE 5
Large branches in motion; umbrellas hard to hold; telephone wires whistle.
25-31 miles per hour.
STRONG BREEZE 6
Whole trees in motion; walking against wind difficult.
32-38 miles per hour.
MODERATE GALE 7
Twigs break off the trees.
39-46 miles per hour.
FRESH GALE 8
Slight building damage.
47-54 miles per hour.
STRONG GALE 9
Seldom happens inland; trees uprooted; much damage.
55-63 miles per hour.
WHOLE GALE 10
Very rare; much general damage.
64-72 miles per hour.
STORM 11
Anything over 73 miles per hour is a
HURRICANE 12

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WINDCHILL FACTOR

Actual Temperature (°F)	Wind Speed 0–4	5	10	15	20	25	30	35	40	Level of Danger That Exposed Flesh Will Freeze
45	45	43	34	29	26	23	21	20	19	Little danger
40	40	37	26	23	19	16	13	12	11	
35	35	32	22	16	12	8	6	4	3	
30	30	27	16	9	4	1	−2	−4	−5	
25	25	22	10	2	−3	−7	−10	−12	−13	
20	20	16	3	−5	−10	−15	−18	−20	−21	
15	15	11	−3	−11	−17	−22	−25	−27	−29	Increased danger
10	10	6	−9	−18	−24	−29	−33	−35	−37	
5	5	0	−15	−25	−31	−36	−41	−43	−45	
0	0	−5	−22	−31	−39	−44	−49	−52	−53	
−5	−5	−10	−27	−38	−46	−51	−56	−58	−60	
−10	−10	−15	−34	−45	−53	−59	−64	−67	−69	
−15	−15	−21	−40	−51	−60	−66	−71	−74	−76	Severe danger
−20	−20	−26	−46	−58	−67	−74	−79	−82	−84	
−25	−25	−31	−52	−65	−74	−81	−86	−89	−92	
−30	−30	−36	−58	−72	−81	−88	−93	−97	−100	

If you have some idea of the wind speed, you can consult this chart to find out what the temperature will feel like when you go outside. Study the wind chill chart with your children and see if you can find a pattern for the way the actual temperatures and the "wind chill" temperatures are related.

RECORDING TEMPERATURES

You probably have an outdoor thermometer. If you don't, and you plan to buy one, try to get one that shows the temperature in both the Fahrenheit and Celsius scales. If your child is interested in keeping track of, and recording temperature changes, a thermometer which shows daily minimum and maximum temperatures would be ideal.

The evening weather news gives the current temperature reading, as well as the high and low reading for that day. In addition, it gives the normal (or average) high and low temperature reading for that particular day and the record high and low temperature with the year this record was set. With a maximum/minimum thermometer, your children can calculate their own average daily high and low temperature for any period of time. They can also figure out the average temperature for a day, a week, or a month. Ask them to take as many different temperature readings during the day as possible. If they want to have an average figure for the day, they have to add all these temperature readings together and then divide that figure by the number of times they read the thermometer. (The figure you get by this calculation is called the "arithmetic mean"; the average temperature is sometimes called the "mean" temperature.)

WEATHER STATISTICS

In Colorado, daily temperature readings are particularly interesting. There are big swings in temperature between day and night and the day to day temperature can also change dramatically. You will get an interesting graph when the temperature changes frequently and shows many ups and downs. If your children are doing any graphing in school, encourage them to keep some weather statistics. The discipline of having to take daily readings is good training and the accumulating information will teach your children a great deal, both about the weather and about graphs and statistics. Averages can be deceptive. Have your youngster figure out the average temperature of a week with widely varying temperatures. Does the average tell you how hot and cold it was?

Individual recordings of different weather measurements become weather statistics when they are kept over a period of time and are then organized in some way. Graphs are a good way to display weather information. They give a visual picture which makes it easy to read and interpret the various weather measurements. Look at your local newspaper and see what kind of weather statistics it publishes. You can also contact the Colorado Climate Center in Fort Collins.[1] The Center collects a large variety of weather statistics for most of the larger communities in the state. Write to them and ask for information on your community, or phone and leave a message and they will call you back.

PROBABILITY

You may not be aware that when you are wondering whether it will rain or snow, or be sunny or cloudy, you are dealing with the mathematical topic of probability. What does it mean when the weather report says that there is an 80% chance of rain, a 40% chance of snow or a 30% chance of high winds for a particular day? The weather service looks at weather patterns like temperature, relative humidity, cloud formation, wind force and direction, barometric pressure and so on. If it has rained on five out of ten days in the past with similar weather conditions then the probability of rain for that particular day will be five out of ten, or 50%. Weather predictions are made on the basis of how things happened in the past, which is one of the reasons why weather bureaus keep weather statistics.[2] If you listen to the weather news with your children and they hear predictions about the chance of rain, ask them how they think the weather forecaster knows. This is a good opportunity to introduce the topic of probability.

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[1] Colorado Climate Center, Department of Atmospheric Science, Colorado State University, Ft. Collins, CO 80523 Phone: (303) 491-8545

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[2] ***Sky Watch the Western Weather Guide*** by Richard Keen is an interesting, richly illustrated and very readable book for adults. It has a good mix of factual information, weather statistics and related anecdotes. Published by Fulcrum Inc., Golden Colorado, 1987. Price was $13.95 in 1988.

15. TIME

THE SEQUENCE OF EVENTS

What does time mean to very young children? They may hear phrases such as "it's time for your lunch...it's almost time for your bath... it's bed time..." and so on. The word "time" gets associated with certain important events which take place during the course of their day and which mark the passing of time.

In pre-school or day care there are certain daily routines which children learn to remember: play time, snack time, outdoor time, lunch time, rest time, story time, going home time. These events form a sequence which helps children to predict what will happen during their school day. Ask your young children questions about their school routines, such as: "What do you do after you have finished with lunch?" If they don't know, you could add: "Is that the time when you get on your mats to rest?" This kind of comment or question helps children to remember how the day progresses.

Children also love to listen to stories about themselves. For instance: "Once upon a time there was a little girl called Jenny. Every morning her mother went to her room and told her it was time to get up to get ready for school. Jenny got out of bed, put on her favorite yellow pants and purple shirt, and then went to the kitchen for breakfast...." The more details the better. You can also ask your children to tell a story about an interesting day, or to draw pictures of a sequence of events.

Using the names of the days of the week, and time words such as "today," "the day before yesterday," "the day after tomorrow," "last night," "next week," and so on will help children to place events into a correct time sequence.

Gradually, the time span that children can remember and anticipate grows longer and important events help to mark the passage of months and years: children's birthdays, holidays celebrated with family and friends, the seasons — remembered through many changes: temperature, length of days and nights, the way trees look after their leaves have fallen off, things children do outdoors like swimming or sledding, and the clothes children wear at different times of the year. Related time words like "winter," "spring," "summer," and "fall," "last year," "this year," "next month," "in two months," gradually take on meaning.

GETTING A FEELING FOR TIME

At first, measuring time is done quite informally in the home. "Let's see how fast you can pick up your toys. I'll start counting slowly"... "You can have five more minutes in the bathtub. When this timer rings, it'll be time to get out"... "Can you set the table before I take the cookies out of the oven?"..."Can you brush your teeth while I finish this phone call?" and so on. Later, children get interested in timing activities with clear beginnings and endings. Use the second hand of a watch or clock to time some of the following activities and encourage your kids to add their own ideas. (A stop watch would make a great gift for children who are interested in measuring time.)

How long does it take you to:

- walk upstairs?
- walk, jump, hop to the mailbox?
- get dressed in the morning?
- tie your shoe laces?
- walk, run, or bike around the block?
- bounce a ball one hundred times?

How many times a minute:

- can you say your name, write your name, jump rope, bounce a ball, run up and downstairs, dial your phone number, or hop?

Anticipating future events:

- how many days before your birthday?
- how many weeks before school is out?
- how many years before you can drive a car?

How many hours a day do you spend:

- sleeping, eating, watching TV, playing, riding the school bus, doing your homework?

If children know how to multiply, give them a calculator and let them figure out how many hours a week, a month, or a year they have spent doing the above things. How many hours in their lifetime have they done a small task like brushing their teeth? How many hours have they slept since they were born? How old are they in minutes? How many years is a million minutes?

MEASURING TIME

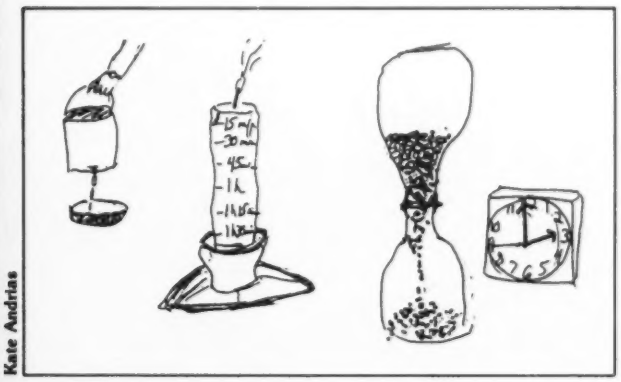
What do you have in your house that you can use to measure time? Clocks, watches, a kitchen timer, an hourglass, a stopwatch or an electronic clock on a microwave? Did you ever wonder how

people measured time before mechanical clocks existed? The cycles of nature gave man their first units of time: sunrise and sunset, the phases of the moon, the tides, the changes of the seasons, the orbiting planets. Later, people invented water, sand and candle clocks to measure smaller units of time. Your children can use their imagination to invent measuring devices based on the following suggestions:

They can suspend a bucket over a bowl, trickling water or sand through a small hole in the bottom of the bucket. Then they can time how long it will take the bucket to empty. This length of time becomes the unit with which to measure other things.

They can measure how far a candle burns down in fifteen or thirty minutes and make a mark. Then mark the whole candle in these units to make a candle clock.

Use a commercial egg timer or make one out of two small plastic bottles with narrow necks. Fill one bottle with salt and time how long it takes for the salt to run into the other bottle.



Kate Andrias

USING REGULAR RHYTHMS TO MEASURE TIME

There is a wonderful story about the famous scientist Galileo who was born in Italy in 1564. Young Galileo was in church watching a large chandelier which had started to swing. Watching the swings he noticed that though the distance covered by the swings decreased, they still all took the same amount of time. He timed the swings using his pulse. This is how the pendulum was supposed to have been discovered.

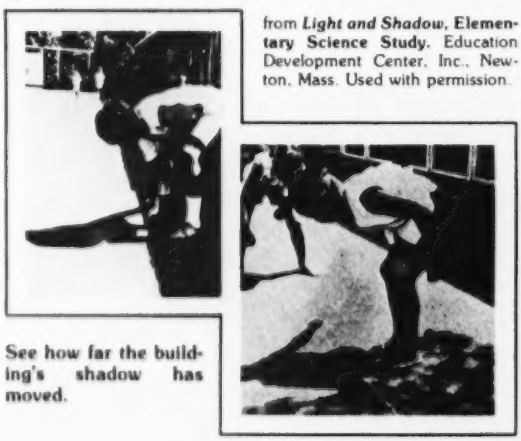
Any weight tied to a string which is suspended from a hook becomes a pendulum when you start it swinging. How fast the pendulum swings back and forth depends on the length of the string. Children like to call a back-and-forth swing a "round trip." (The physics term for a round trip is "period.") Even though the distance a pendulum covers on its round trips after you start it swinging gradually becomes shorter and shorter until all motion stops, the length of time it takes for each round trip never changes. That was Galileo's great discovery. Grandfather pendulum clocks were built on that principal.

Children enjoy playing with pendulums and counting a certain number of round trips will give them a reliable time measurement which they can use to time other things. You could also use a metronome, a regularly dripping faucet, your own pulse, or flashing lights to time some other event. Ask your kids if they can find such "timers" around the house or neighborhood.

SUN AND SHADOWS

Shadows change as time passes; that's the reason sundials work. Before children can understand sundials they must have lots of experiences with shadows. Draw your children's attention to the shadow of your house, of a tree or fence in your yard, or of a nearby lamp post. Look at these shadows in the early morning and late afternoon, at noon and at different times during the day. See if your children notice how the length and the direction of the shadow changes. When is the shadow the longest? The shortest? When is it the same length as the object that casts the shadow?

After lots of shadow watching, you can make a simple sundial (sometimes called a shadow clock) by tracking the shadow of a stick, a fencepost, a mailbox or any other object that is convenient.



from ***Light and Shadow*, Elementary Science Study.** Education Development Center, Inc., Newton, Mass. Used with permission.

See how far the building's shadow has moved.

Trace the shadow with chalk or place markers at the end of the shadow. Do this at hourly intervals and write down the time of day by each shadow line. Every sunny day thereafter, you can read the time off these markers or lines. There are children's books on shadows which will give you directions for making more permanent sundials.

LEARNING TO TELL TIME

I have purposely put this section at the end of the chapter because children need many time experiences before telling time from a clock makes any sense. Telling time is an important skill and children derive great pleasure from mastering it, yet often this subject is introduced too early in school. Some first or second graders really struggle with learning to tell time, when a year or two later they might learn to do it with ease. Try to take your cues from your children: are they asking you questions about time? Do they want you to show them on clocks what time it is? If so, by all means help them. Pick up an old wind-up alarm clock at a garage sale and let your kids play with it. Place a digital clock next to a circular clock and encourage them to observe the changes. Show them how the large and small hands on clocks work together, teach them about half and quarter hours, let them set the kitchen timer for five or ten minutes and have them watch how far the big hand on the clock moves during that time.

When they begin to understand clock time, let them practice with simple calculations. Here are some suggestions:

Look up when a TV program or a movie starts and ends. How long does it last?

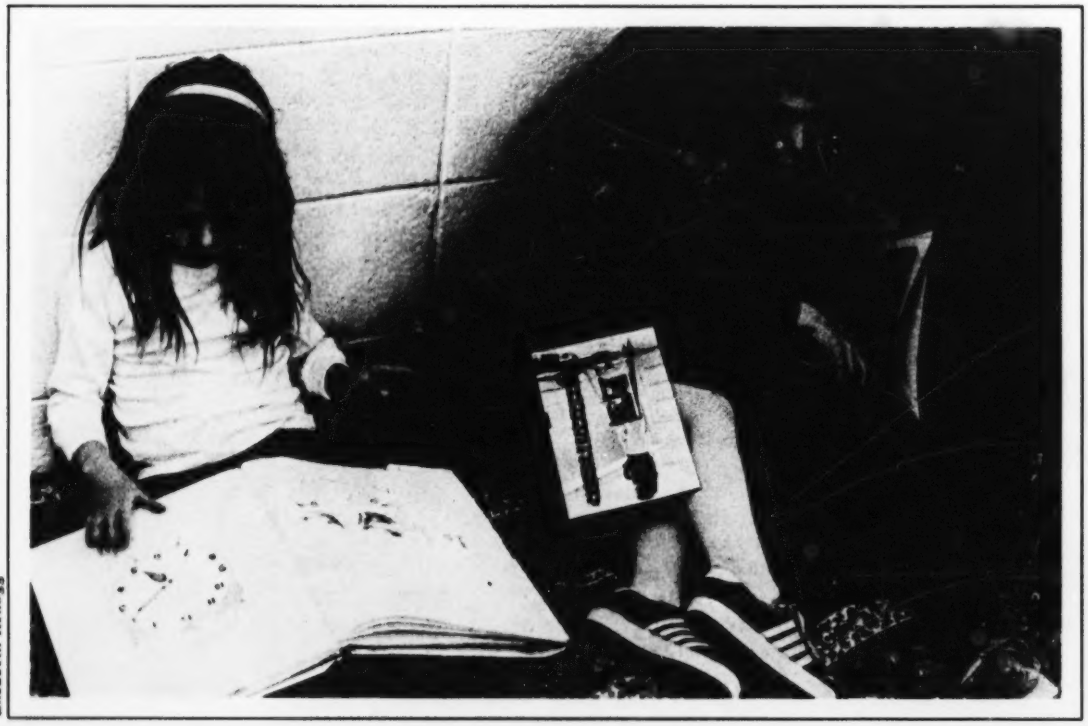
If you sleep ten hours at night and have to get up at six, when should you go to bed?

If you put a quarter in the parking meter now, at what time will it expire?

Dinner will be ready in half an hour. What time will it be?

Older children can figure out what time it is in different cities in the United States and in other parts of the world.

Try to remember that almost everyone learns to tell time from clocks sooner or later but many people go through life without ever developing a good sense of time. I hope this chapter has convinced you that there is more to find out about time than just to learn to read a clock!



Elizabeth Kellogg

MATHEMATICAL NOTES

1. LEARNING TO COUNT

LEARNING THE NUMBER WORDS

The ability to count develops gradually over a period of years. You can observe the stages in which this happens in your own children. As children hear number words being used (one, two, three, and so on), they repeat them the way they repeat all words they hear. Soon they try to recite the number words in the sequence in which they hear them but it takes quite some time before they can always say them in the right order. At first number words are just like nonsense syllables — children have not yet connected them with the numbers which they identify.

To help your children learn the number words, you can count out loud with them while you or they engage in an activity that has a regular rhythm. Hammering nails, bouncing a ball, turning a crank, pumping air into a bike tire, walking up or down some steps, jumping rope or dealing cards can all now and then be good "counting" activities. The aim of this kind of counting is not to find out how many times you hit the nail or turn the crank, but rather to use the activity to count out loud so that your children can hear the words repeated in the right order and can learn them by counting along with you.

USING THE NUMBER WORDS TO COUNT OBJECTS

Children eventually begin to use the number words they have memorized to count objects. It takes children a long time to understand that to count things, you match each number word with exactly one of the objects you are counting. (Counting is a special kind of matching.) Watch your children when they are beginning to count objects: you may notice that although they are touching the objects they are counting, cookies for instance, they are not necessarily matching one number word with only one cookie. They may skip some cookies and count others twice, because they do not yet understand that each cookie may be touched or counted only one time.

COUNTING FROM THE CHILD'S POINT OF VIEW

Imagine that you are learning to count. You match the number words you have memorized to some raisins. As you touch the first raisin, you say **one**, as you touch the second raisin, you say **two**, and so on. From the child's point of view, this could be a mysterious process. You go through the identical motions as you touch the objects to be counted yet each time you make that motion, you have to say a different word.

Children have to learn another very strange thing: when you say **two**, the preceeding **one** is part of the **two** and when you say **three** the preceeding **one** and **two** are part of the **three**. If you are counting six raisins, you must understand that raisins number **one through five** are included in the number **six**. The position of raisin number six in the total collection of raisins tells us that five raisins have already been counted. If you eat raisin number five, raisin number six becomes raisin number five.

Many young children at first think that number words are **names** for the objects being counted, just as Carol and Kenny are names of people. Carol and Kenny are always called by the same name, regardless of whether they are in a small group, in a long line or by themselves. If you have a bunch of colored blocks you can make all kinds of patterns using different numbers of blocks. Yet the same red block may be number two in one arrangement, number five in another one and number eleven in a row. Children have to learn that the number words are not names for these blocks: they simply tell us **how many** blocks there are in this collection.

These are very difficult ideas for young children to grasp. (They may be difficult for many adults, too, who think about them for the first time.) Only time and repeated experiences with counting objects will help them come to a full understanding.

UNDERSTANDING NUMBERS

To understand counting fully, children must also learn the following:

1. The order of counting does not matter. If the objects are lined up, you can start at either

end or in the middle, as long as you count every object only once.

2. The appearance of the objects to be counted does not matter. Objects can be pushed together into a pile or spread out, arranged into rows or circles or scattered randomly. If there are eight objects to be counted, there will always be eight objects, no matter how much you move them around.

3. Groups of very different things can be counted. Children generally start out counting groups of identical objects. Later they learn that collections of objects having nothing to do with each other can be counted too. What do the wheels of a car have in common with a group made up of an elephant, a banana, a marble and an aspen tree? There are four things in each group.

Learning to count correctly is a process that can take a long time. Enjoy your children's progress, however slow it may seem to you, and at any one time accept their ability to count the way you accepted their ability to walk and to talk. Don't be in a hurry. With lots of practice and with your encouragement, your child will learn to count with confidence and enjoyment.

2. MEASURING

LEARNING TO MEASURE

Just as you can learn to count by memorizing the number words without really knowing what they mean, so you can learn to measure by using the numbers on rulers or scales without understanding what measurement is all about. Children need many experiences before they are ready to use standard measuring units.

When you count objects, you are dealing with separate, individual things. You count to find out how many things there are. When you measure, you are dealing with a continuous quantity like a length. You are still counting, only now you are counting units of measure rather than objects. You want to find out how many times a unit of measure (a standard unit like an inch or a non-standard unit like a toothpick) will fit into the object you are measuring. "How many of these (your units) make one of those (the thing you want to measure)?" When you ask: "How long is my couch?" you want to know how many inches, end to end, can be lined up along the couch. When you ask: "How heavy is this box?" you want to know how many pounds will equal the weight of this box.

Everything we measure, whether it's length, area, volume, weight, temperature, or time, has its own units of measure. These units have different names, such as inches, pounds, degrees or minutes, but the process by which we measure is always the same: we compare the units to what we want to measure.

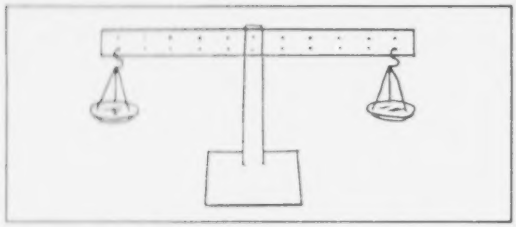
There is a sequence in which most children learn to understand measurement. It starts with making simple comparisons of two objects: which piece of yarn is longer, the red one or the green one? Which of two packages is heavier? Which glass has more water in it? Which one of us is taller, which shorter?

When children have done lots of comparing of two objects or quantities and feel comfortable with this idea, they can start to compare more than two things, putting them in order by length, height, weight, or some other attribute. For instance, they can line up three pieces of string from the shortest to the longest; they can put five glasses of water in order from the one with the least amount of water to the one with the most water; they can arrange any number of cans of food ordering them by height or by width. When they can compare the sizes of objects and also know how to count, children are ready to begin measuring.

Beginning measuring activities should be done with so-called **non-standard** measuring units. (Some common **standard** units are inches, yards, ounces, pounds, quarts, and so on, as well as their metric equivalents.) **Non-standard** units (like straws or marbles) will help children to focus on the process of measuring rather than on learning to read the markings on rulers or scales without knowing what these markings mean. In the beginning, it is enough simply to see how many straws, lined up end to end, will be as long as the object being measured or how many marbles will equal the weight of a book. Later, children will learn to work with standard units.

Look for non-standard units around your house. For **length** measurements you can use any set of equally long objects: straws, toothpicks, pieces of rope cut to equal lengths, strips of wood, whole pencils or crayons, as well as parts of your body.

For measuring **weight**, you'll need some sort of balance — a simple homemade one will do. (see illustration) Pebbles of approximately equal size, beans, washers of various sizes, or small containers such as film cans filled with sand all make good units for weighing things.



For measuring the **volume** of a box, small wooden cubes to pack inside are ideal but sugar cubes work well too. You can use rice to measure the volume or **capacity** of a jar, (see footnote, p.48) using scoops or jar lids as units of measure. Fill your unit with rice, pour it into the jar and then count how many units it takes to fill the jar to the top.

For taking **area** measurements, find something with which you can cover a surface like a tabletop, the kitchen counter or a rug. Napkins, sheets of typing paper or paper towels will do the job.

When adults measure the length of a long table, they may take a twelve-inch ruler and move it along the table's edge, counting the number of times the ruler is moved. Eight moves, for instance, will make eight feet or ninety-six (8x12) inches. Young children often do not understand this method. They need to see their units (straws, for instance) lined up along the entire length of the table, however tedious this may seem to an adult. It takes a lot of experience before children realize that they can move their unit along. For that reason children will need large quantities of their non-standard units when they begin to measure. (You could hook together a string of paper clips or safety pins for a useful early "tape measure.")

Give your children many opportunities to measure with non-standard units. They will enjoy telling you that the dining room table is seven straws wide or fifty toothpicks long, or that the bag of oranges weighs forty pebbles or twelve large washers. Encourage them to find their own measurement units after you have introduced them to this way of measuring. Choosing their own units will add to their pleasure and their learning.

If at all possible, try to let children discover for themselves why measuring units were standardized. Let them see what happens if, for instance, both you and your child use baby steps (heel to toe) to measure the length of a room. The number of footsteps taken will vary according to whose foot you use as a unit. You'll run into similar difficulties if you measure a tabletop with hand spans, fabric with arm spans, or the backyard fence with whole bodies (which children love to do). Is the fence 15 Marys, 12 Tommies, or 8 daddies long? When children begin to realize what problems can arise when non-standard units are used, they may begin to understand why standardized units were adopted and will appreciate learning about them.

If children already understand the principles of measuring, they will quickly learn to substitute the standard units for the variety of units they have been using previously. It is important that children experience measuring with both the U.S. and the metric standards. Children who have worked with non-standard units accept the metric system with little trouble. Having invented their own units, they know that units come in many sizes and can exist side by side.

THE METRIC SYSTEM

The key to becoming more comfortable with metrics is to stick to one system at a time. Don't try to go back and forth, converting numbers from one system to another. It won't take you long, for instance, to get a feel for temperature measurements in Celsius if you have a Celsius thermometer around your house. On a very hot day, your thermometer may show 35 degrees. You'll remember that 35 degree is hot without referring to the Fahrenheit scale!

It is also helpful to visualize everyday objects which are measured metrically: a one-liter soft-drink bottle; 35 millimeter film (it is 35 mm wide); 150 centimeter long skis, and so on. A meter (m) has 100 centimeters (cm) and one centimeter has 10 millimeters (mm).

Here are some other useful benchmarks:

- The freezing point of water is 0° Celsius.
- The boiling point of water is 100° Celsius.
- Comfortable room temperature is 20° Celsius.
- A liter is a little more than a quart.
- A kilogram is a little more than two pounds.
- A meter is a little longer than a yard.
- An inch is about 2 1/2 centimeters.
- A foot is about 30 centimeters.
- 8 kilometers is about 5 miles.
- 88 kilometers per hour is about 55 miles per hour.

MEASURING LENGTH, AREA AND VOLUME

Children generally begin their measurement activities by measuring different **lengths**. When we measure length, we take measurements that go in one direction only. There are many things we measure in one direction, or in one **dimension** — the term used in geometry. Width, depth, height, thickness or distance are all measurements of length. If you have taken many lengths measurements, you get to be pretty good at making estimates.

Taking measurements for an **area** is almost as simple but understanding what is done with these measurements is more complicated. If you want to know the area of your kitchen floor because you plan to get new vinyl tiles, you take **two** measurements, length and width. You are measuring in two directions now, for example north/south and east/west. You multiply these two lengths to find out how many square feet would cover the floor. A one-foot square is a unit of measure for area. It is called a **square foot**. If your kitchen is 7 feet wide and 9 feet long, it has an area of 63 **square feet**. People who deal with square foot measurements in their work like architects, real-estate brokers, painters or carpet-installers become very good at making estimates of area in square feet.

You measure the **volume** of three-dimensional objects which take up or enclose space. To get a volume measurement, you have to measure in three directions such as north/south, east/west and up/down. Then, if the object is box-like, you multiply these three measurements to find the number of units of volume. Units of volume are called cubes – for example a one-foot cube (cubic foot – cf) or a one-centimeter cube (cubic centimeter – ccm). A refrigerator whose storage space is two feet wide, two feet deep and four feet high has a **capacity**[1] of sixteen **cubic feet** (2 x 2 x 4). People in the building trades who pour concrete and order bricks or sand, furniture movers who know the inside space of their vans, as well as gardeners who order manure or top soil, are all familiar with cubic measurements.

Volume can describe both how much space is inside something and how much space something takes up. When you measure the storage space in your refrigerator, you are measuring the space it **encloses**. If you wanted to know how much space the refrigerator **occupies**, you would have to measure and multiply its three outside dimensions. To help children understand this idea, you could take a plastic measuring cup and fill up the entire interior space with modeling clay. Then pry out the clay carefully, so it will keep the shape of the cup. The lump of clay and the empty cup will have the same volume.

It is important to expose children to many different experiences with area and volume measurements to help them to become familiar and comfortable with these units. Many well educated people are quite lost when they have to give estimates of area or volume!

A SURVEY OF MEASURING INSTRUMENTS

With older children it might be fun to start thinking about measurement by taking an informal survey of measuring instruments which you already have in your home. If you have a garage, a shed, or a workshop in the basement, be sure to include these too.

You could start out by making a list with your children: what kinds of measuring instruments do they have in their rooms? What kinds of devices can they find in the kitchen, bathroom, or basement? They will probably come up with the more obvious things at first, like rulers, thermometers, measuring cups or bathroom scales. As you and your children begin to think more about measurement, you may find all kinds of other equipment which you had not thought about before.

A measuring survey can be a fine family activity which you can stretch out over many weeks. You could start a measurement book in which family members record measurement instruments they see at home, at work, or at play. If your list gets to be long, you can make up different categories, classifying your instruments by what they measure — length, volume, weight, time, temperature and so on. Once you get started on your survey you may be surprised to find how measurement pervades all aspects of our lives.

3. DISCOVERING PATTERNS

A number of the activity chapters referred to mathematical patterns: patterns made by decorating cakes or by arranging foods for a special occasion; patterns created in knitting, weaving and craft projects; leaf patterns of growing garden

[1] The volume measurements of space to be filled like the space of a refrigerator, a bath tub or a cooking pot, are often referred to as **capacity**.

plants. Patterns can be found everywhere: in art, music, and language; in architecture and design; in the natural and the man-made worlds; in the rhythm of our daily lives as well as in our behaviour; and, last but not least, in numbers and in shapes. We use, record, search for, and create patterns. We find pleasure in their regularity and rely on their predictability.

In many kindergarten classrooms, children are encouraged to make, look for, and listen to patterns. They are given all kinds of materials to create patterns with color and shape: they string beads, make long rows with different shapes of wooden blocks, or design patterns with crayons,

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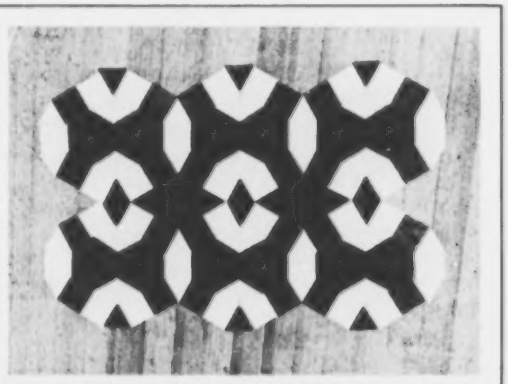
yarn or different types of seeds. They learn about musical patterns by clapping rhythms. Children also learn how the same pattern can appear in different forms: They may clap this rhythm: ·· ·· ··, string a necklace of red-blue-blue-red-blue-blue-red-blue-blue beads, look for the same pattern in the syllables of children's names, like Jennifer, Annabelle, or Ferdinand, or line up in a pattern of tall-short-short-tall-short-short-tall-short-short children.

If parents wonder why their children are doing these things they may be told that working with patterns prepares children for learning mathematics. This answer may not always be fully understood. What is the connection between stringing beads and learning math? Becoming aware of patterns in their everyday life or creating patterns often helps children to look for and to recognize the countless patterns that exist in numbers and in shapes. Here are some examples of simple but important mathematical patterns found in beginning number work.



Tony Kallet

This pattern was made with black and white triangles, squares and hexagons cut out of linoleum tiles.

NUMBER PATTERNS

Think of how numbers are written: Beginning with 0, we use a different symbol for each of the first ten numbers: 0,1,2,3,4,5,6,7,8,9. Then each of these symbols is repeated following the number 1: 10,11,12,13,14,15,16,17,18,19; then again following the number 2 and so on. As soon as a child catches on to this pattern, writing numbers becomes much easier.

Can you see a pattern in this list of numbers? 1,3,5,7,9,11,13,15,17,19,21,23,25,27,29... Some people first notice that these are all odd numbers. Others see that each one is two more than the one before. Still others notice that the right hand digits repeat the sequence 1,3,5,7,9, over and over.

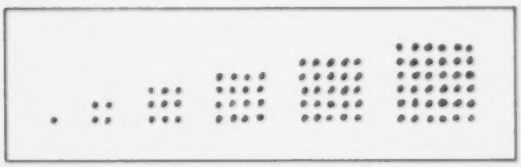
Here are some addition and multiplication patterns. 7+8=15 and 8+7=15. 2+4=6 and 4+2=6. Children who do not see this pattern must learn twice as many sums as children who see and use the pattern.

The same rule holds true for multiplication. 6x9=54 and 9x6=54. Understanding this pattern cuts in half the numbers that have to be memorized in the multiplication tables.

How many patterns can you find in the 9's table? Write down the numbers (9, 18, 27, and so on) in a column and look at them carefully. The longer you look, the more patterns you will notice.

Recognizing number pairs that add up to 10 (1 and 9, 2 and 8, 3 and 7 etc.) and later to 100 (45 and 55, 60 and 40, 91 and 9 etc.) is also a useful skill, especially for adding up columns of numbers and for making quick estimates.

Here is an interesting series of numbers. 1, 4, 9, 16, 25, 36, 49, 64, etc. Cover the rest of the page while you think about it. Maybe you saw that the numbers go odd, even, odd, even, odd, even, etc. Or you noticed that they increase first by 3, then by 5, then by 7, and so on, and that these are all odd numbers. Did you realize that all the numbers in this series come from numbers that were multiplied by themselves? 1x1=1, 2x2=4, 3x3=9, 4x4=16, 5x5=25, and so on. These numbers (4,9,16 etc.) are called **square** numbers because 3x3 can also be written 3^2 which is read "three squared." They are called square numbers because in geometry, they can be arranged into squares.



Try making a square out of 7 or 12 or 20. It won't work because only numbers multiplied by themselves will make squares. That's why they are called square numbers!

One of the most famous arrays of numbers in history is called Pascal's Triangle. (Pascal was a French philosopher and mathematician who lived in the 17th century. He did not invent the triangle but he wrote a famous book about it.)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
and so on

There are literallly dozens of patterns in this triangle. Can you find the pattern which will tell you how to continue the rows?

Searching for patterns can be very absorbing. If your children are intrigued by number patterns, you and they can make up lists with secret patterns to be guessed. Don't be too surprised if others find patterns in your list which you didn't know were there!

As children progress in their mathematical studies, patterns assume an ever more important role. In work with fractions, in algebra, geometry and calculus, an understanding of the relationships between numbers is crucial. Children who learn to see patterns in numbers in their early school work are much more likely to enjoy mathematics and to be successful in their studies.

Discovering number patterns can be deeply satisfying. It often gives children confidence because pattern reveals some of the underlying structure of mathematics and shows that numbers behave in a predictable way.

PATTERNS IN NATURE

The mathematics of nature has fascinated both scientists and non-scientists. The lay person is

Julie King

Julie King

probably most impressed by both the abundance and the beauty of shapes and patterns that can be found in plant and animal life, as well as in inanimate matter like rocks and crystals. Scientists and mathematicians marvel at the relationship of form and function, observing how the shapes of animals and plants have helped the various species to survive. They study the geometry of nature and have discovered some intriguing number patterns in the arrangements of flower petals and seeds and in the growth patterns of tree branches and snail shells.

Here are some examples of shapes and patterns that you can discover with your children. Enjoy them for their own sake but remember also that finding geometric shapes in nature will help children to see and value mathematics outside of the classroom.

Round shapes are very common: there are circles like wave patterns in water hit by a stone or rings in the trunks of trees; and there are many spheres (three-dimensional circles) like puff balls, dandelion heads, oranges, and the huge spheres of the planets in our solar system. (Some are, of course, only approximately spherical.) There are also circular arrangements like flower petals, apple seeds (if apples are cut crosswise) and the rolled up bodies of hibernating animals.

Trees and some leaves have **triangular shapes** as do fish scales and a few species of fish. Snowflakes are **hexagonal (six-sided)** and so are the compartments in the bees' honeycomb. Many flowers have five petals and star fish and sea urchins also show divisions by five. (A shape with five sides is called a **pentagon**.)

Spirals occur frequently in nature. Look for spiral patterns in spider webs, in many sea shells, or in the horns of certain sheep. The elephant curls his trunk into a spiral and the chameleon creates a spiral with his tail. **Helixes** are three-dimensional spirals and can be observed in whirlpools, tornados and corkscrew shells.

Symmetry is abundant in the natural world: butterflies and insects, leaves and pairs of animal footprints all have mirror symmetry where the left and right sides are identical. Flowers like daisies and asters, fruits like oranges or tomatoes if cut crosswise, as well as six-sided snowflakes all have rotational symmetry: turn any one of these a little and it will look the same.

The above examples just begin to hint at the richness of nature's shapes which you can explore with your children. If this topic interests your children, you can order an excellent little book called ***Nature Takes Shape*** by Dorothy and John Paull.[1] For older children and for yourself, watch for a repeat screening of a fascinating Nova Show called "The Shape of Things" which also deals with this topic.

4. COLLECTING STATISTICS

My dictionary defines statistics as "the mathematics of the collection, organization, and interpretation of numerical data." Data (which, by the way, is the plural of datum but is frequently used in the singular) simply means bits of information. Often you collect data by making surveys. For instance, the Park Service may want to know how many people visit the Rocky Mountain National Park each year and how they spend their time there. Such information would help them to make plans for the future. Or, the Social Service

[1] ***Nature Takes Shape*** is published by Ladybird Books Inc., 499 Omni Circle, Auburn, Maine 042210. Price was $2.50 in 1988. Ask for their catalog to see other books about mathematics.

Department in Grand Junction wants to know the average monthly income of single working mothers because it must make some budget decisions about day care facilities. When you do statistical analyses, whether you are a third grader, a social worker, or a scientist, you start out by collecting data about something that will be answered with numbers: how many, how often, how long ago, how much, how fast, and so on.

After collecting your data, you organize them so that it will be easier to figure out what they mean. The Park Service statisticians may list the numbers of people who visit the park in each month of the year. They may want to know where visitors come from and how long they stay in the park; they may have categories for campers, hikers, backpackers, and motorists; a breakdown for their age and sex; and a record of attendance at park-sponsored activities. This will be their data. The Social Service Department researchers may want to know the age of the working mothers, how many children they have, how old the children are, whether mothers work full time or part time, and who takes care of the children when they are not in school. Tables, charts, and a variety of graphs are the most common ways in which ordered and organized data are displayed. Now the information that has been gathered can be analyzed and interpreted. For instance: Are there so many visitors to the Rocky Mountain National Park in the summer months that shuttle busses should be used to transport people to the most popular spots? Should there be more campgrounds? More programs for older people? Or: What would be the most economical way to care for the many pre-school aged children in Grand Junction who need daycare facilities so that their mothers can go to work? Should employers contribute to the cost of day care? Could older citizens' skills be used in some ways? Such questions may arise after the collected statistics have been studied and analyzed.

Statistics are used frequently by people in many occupations. We are bombarded with statistical information in newspaper and television ads, in government and medical reports, in weather and sports news. It is important that we learn to understand statistical information so that we can draw our own conclusions as knowledgeable citizens and consumers.

Many examples of statistics are given in this booklet, especially in the chapters on Personal Statistics and Weather. Sports is another topic through which children can learn some of the basic data collecting and organizing skills. Sports statistics are published in most daily newspapers. If your children are interested in sports — and many elementary school children are — you can help them find and read the sports statistics and perhaps show them how they can make their own tables and graphs.

Collecting statistics about a subject that interests them is just one more way in which children can learn about the importance of mathematics in their lives.

5. SOLVING PROBLEMS

You may have heard your child's teacher talk about the current emphasis on problem-solving skills in mathematics instruction. What exactly are problem-solving skills?

In our day-to-day lives, we are confronted with problems all the time. "I've got a problem with my car. The alternator light keeps coming on but my mechanic can't find the reason." "We have a problem taking our family vacation. Johnny is playing in a tennis meet and doesn't want to miss it..." "We found the perfect house but can't buy it till we sell our old one..." How do we solve such problems? There is no easy answer, no formula which we can apply, no quick way to dispose of such problems. We have to **think** about them, look at them from different points of view, compare possible solutions. You undoubtedly solve problems all the time, using many different approaches.

A problem in mathematics is not very different. It is generally stated in the form of a question for which there is no obvious answer or way of getting an answer. You can't just go through some mechanical process, like applying a formula you have learned. You have to think first whether there is a formula that would work and try many different approaches to arrive at a solution.

There is no special mathematical topic called "problem solving." Problem solving skills are **thinking** skills in mathematics as well as in the rest of life.

This book is full of problems for you and your children to think about. In every chapter there are suggestions and questions about your family, your home, and your daily activities which require problem-solving skills to find answers.

Pre-schoolers are solving problems when they learn to match and sort objects. Grade school

children are solving problems when they figure out whether a bath or a shower uses more water. You are solving a problem when you decide how long to bake a casserole if you double the recipe. Whether a mathematical question is a **problem** for you depends on your age, your experience, and on how much math you know.

When you hear problems mentioned in connection with mathematics, you may be thinking of "story problems" which can be found in most math textbooks. Many people have unpleasant memories about these paragraphs that were supposed to help you apply the math you had learned in real-life situations. Many of these story problems have nothing to do with the real world of children. Moreover, they don't challenge children's thinking if all the problems on one page can be solved in the same way, by applying the mathematical procedure just learned. Here is an example of such a textbook problem:

"Mary's mother sent her to the store to buy milk, cookies, and orange juice. She gave Mary a $10 bill. Mary paid $2.05 for the milk, $1.89 for the cookies, and $1.15 for the orange juice. How much change did she bring back?"

Textbook writers include such problems to give children practice with using addition and subtraction. They hope the children will know that they should add the three prices and subtract the result from $10.00. In real life, of course, Mary would just take the change out of her pocket and count it.

There is nothing wrong in using situations from the out-of-school lives of children in which they can apply the mathematics they have learned in school. Unfortunately, children often regard problems found in textbooks as boring, tedious and artificial because they cannot make the connection between these problems and their own lives. You can help make this connection. You don't have to make up problems because you can use the many real situations in which mathematical problems arise. At the supermarket you can hand your child the change and the cash receipt and ask her whether you got the correct change. You can ask how many six-packs of soda are needed for the fifteen people expected at the birthday party or how many days you should stop the newspaper delivery if you will be gone from July 3 through July 21. Such problems are real for both parents and children.

Today, many schools are teaching children strategies for solving problems. These are **thinking** strategies that are useful for people of all ages when they are confronted with a mathematical or logical problem. Here are some strategies that can be used with sample problems.

Use real objects to represent the problem:

Melinda was collecting shells at the seashore. She had 8 shells in her bag. Her friend Tommy gave her 5 more. On her way home she dropped 3, but then she found 4 more. How many shells did she have when she got home?

Act it out:

There are 6 people at a party. If everyone shakes hands with everyone else, how many handshakes will there be?

Draw a picture or a diagram:

John, is 4'8" tall. He is ten. His sister, who is twelve, is 5'3" tall. If she grows 3" in the next two years, how much would he have to grow to catch up with her?

Here are some more strategies:

- Look for patterns which may give you a clue.
- Replace large numbers in the problems with smaller ones.
- Make a list or table to organize the information.
- Try to remember a similar problem you have solved before.
- Try trial and error – also called "guess and check."
- Find missing information in a book or ask someone.
- Cooperate with other people to solve the problem.

Some of these strategies or methods may sound like cheating because you probably were not allowed to use them when you were in school. Be assured that they are not. Mathematicians use many of these methods to solve problems.

Children quite often figure out the anwer to a problem but they may not use the method they have learned or which the teacher expects. Encourage your kids to invent their own methods. Many students who excel in mathematics in later years could be spotted by their unconventional approach to solving certain problems when they were younger.

Once children have been introduced to solving problems using a variety of strategies, they enjoy making up problems for others to solve. This can

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become a good family activity. Many families also enjoy doing puzzle-type problems together, including picture puzzles, and there are numerous group games, commercial games, and pencil-and-paper games which require thinking. (See list of games in ***Appendix***.) When playing games with the whole family, children learn that their own ideas can often be as good as those of people who are older and who know more. Family problem-solving discussions can be among children's best memories as they grow up.

One of the most important things that you can do to help your children in school is to encourage their creative and logical thinking.[1] If children know that their thoughts and ideas are valued even when they are not correct, they won't be afraid to tackle new problems when they are not sure of a solution. Problem solving then becomes an exciting challenge rather than a chore.

6. USING CALCULATORS

Throughout this book there are phrases like: "Take your pocket calculator along when you...." "Give your children calculators to figure out...." Parents often worry that if children are given calculators in the early grades, they will not learn arithmetic. By using calculators at appropriate times children can learn arithmetic faster and with more enjoyment. But they must still have a good command of the facts of arithmetic. You cannot use a calculator effectively if you don't know how to estimate the size of the answer and

[1] ***The Book of Think*** by Marilyn Burns offers a large variety of different problems which would appeal to upper elementary and junior high school children. It is published by Little Brown & Co., as are the other two books by Marilyn Burns mentioned in the bibliography.

to do that, you must be proficient at mental arithmetic.

Calculators can relieve us of a great deal of tedious paper-and-pencil work. There is, for instance, little reason why children should struggle with endless pages of long division. They must, however, know how to **do** long division and understand the principles involved; they must be able to do some rough calculations in their head and spot any obvious mistakes they may have made with their calculators. Children who are adept at mental arithmetic will know when they have pushed a wrong button and won't accept a nonsensical answer just because it appears in the calculator's window. They may also discover that there are times when the mind can beat the calculator.

Calculators are labor-saving tools which take the tedium out of calculations. Children should learn to use them so that they can concentrate on more important aspects of mathematics.

For instance, children can use calculators as tools to explore numbers. They will enjoy their explorations and in the process gain valuable insights into arithmetic patterns. By playing around with a calculator children may discover that the answers to some problems are simple whole numbers while the answers to others fill up the whole display, and contain a decimal point. Entering [20] [÷] [5] [=] gives you 4, but entering [20] [÷] [7] [=] gives 2.8571428; 5 is a divisor of 20, but 7 is not. By experimenting with their calculator, children could find all the divisors of 20.

Learning to multiply numbers by 10, 100 or 1000 can also be done with a calculator. Since children can do these multiplication examples quickly, they can concentrate on the results rather than on the computations. They may soon notice a pattern: multiplying by 10 adds a zero to the number, multiplying by 100 adds two zeros, multiplying by 1000 adds three zeros. This is an exciting discovery for many youngsters. Once they have discovered this pattern they may go on to multiply a number by 30 or 500 or 8000, confident that the pattern will be similar.

Very young children may see calculators as just a device for showing numbers, not understanding that they can perform additions. If a child understands the concept of addition you can demonstrate how to add 1 by entering [+] [1] [=]. As these three keys are pressed repeatedly, the calculator will "count." Children love to make it count as high as possible. While they do this, they are learning what the numbers look like. By pressing [2] instead of [1], children can make the calculator count by twos. Starting with an even number, it will show even numbers; starting with an odd number, it will show odd numbers.

Children may like to make the calculator count by other numbers, such as threes, or fives, or tens. This is good preparation for multiplication. Starting with [0] and adding [5] six times brings you to 6 times 5. Later you can show your kids that the calculator has a special key for calculating 6 x 5 more quickly. Older children may discover other patterns as they become familiar with their calculators' functions.

The use of calculators also allows children to experience the excitement of real-problem solving at a time when their knowledge of arithmetic is still rather limited. Many children, for instance, are intrigued with large numbers, love to write them down, but cannot yet add or multiply them. But if they know what addition or multiplication is, they can use the calculator to solve problems involving large numbers. The kind of estimating problems in which you do a lot of multiplying to get to a larger number (see ***Restaurants***, p.29) or the kind of problem where you want to find out how many times your heart beats in a year (See ***Personal Statistics***, p.8) wouldn't be much fun if you had to solve them with paper and pencil. When children don't have to worry about the details of their calculations, they can concentrate on strategies for solving the problem.

There is another reason for using calculators: it puts you and your children on a par while solving problems. Trying to answer interesting questions can become a cooperative project. When a child realizes that she can do computations as fast as you can, she may become more adventurous in looking for strategies and trying them out.

Calculators are here to stay. They are used by almost everyone, in personal as well as in business computations. You probably use one yourself. Calculators are becoming less and less expensive and may soon be as common a tool as pencil and paper. It makes sense to explore the new possibilities which they open up and to make the most of them.

EXAMPLES OF COMPUTATIONS

1. DO I MULTIPLY OR DIVIDE?

This question can often be decided by asking yourself whether you expect the answer to be larger or smaller than what you started with.

Example 1: How many feet in 28 yards?
Feet are shorter than yards.
It takes more feet than yards to measure a length.
The answer should be more than 28.
Multiply 28 by 3. **Answer:** 84 feet

Multiplying something by a number larger than one gives you a larger result.

Dividing something by a number larger than one gives you a smaller result.

Example 2: Raffle tickets cost $1.25. How many can I buy for $20?
If they were $1 each I could buy 20.
$1.25 is more than $1 so I can buy **fewer** than 20.
Multiplying by 1.25 would give me **more** than 20.
Divide 20 by 1.25
Answer: 16 tickets

Multiplying by a number smaller than one gives you a smaller result.

Dividing by a number smaller than one gives you a larger result.

Example 3: It takes 3/4 of a yard of ribbon to trim a potholder. How many yards will I need to trim 12 potholders?
I know it will take **less** than 12 yards.
3/4 is smaller than 1, so multiplying by 3/4 will give me a number **less** than 12. That's what I want.
Multiply 12 by 3/4, or by .75
Answer: 9 yards

2. AVERAGES

To find the average of a group of numbers:
1) Add them.
2) Divide by the number of numbers.

Example: The heights of people in my family are 43 inches, 54 inches, 66 inches and 73 inches. What is our average height?

$43 + 54 + 66 + 73 = 236$

$$4\overline{)236}\quad 59$$

The average height of people in my family is 59 inches.

3. FRACTIONS AND DECIMALS

Here are a few hints and comments which may make it easier to handle computations with fractions.

Finding a fraction of a number:
To find 1/3 or 1/5 or 1/9 of a number you simply divide the number by 3, 5, or 9.

1/3 of 24 is 24 ÷ 3, or 8

1/5 of 20 is 20 ÷ 5, or 4

1/9 of 45 is 45 ÷ 9, or 5

To find 2/3, just find 1/3 as above, and multiply by 2.
1/3 of 24 is 8, so
2/3 of 24 is 2 x 8 or 16.

To find 3/5, first find 1/5, then multiply by 3.
1/5 of 20 is 4, so
3/5 of 20 is 3 x 4 or 12.

To find 7/9, first find 1/9, then multiply by 7.
1/9 of 45 is 5, so
7/9 of 45 is 7 x 5 or 35.

Changing a Fraction to a Decimal:
The easy way to change a fraction to a decimal is to divide the bottom (denominator) into the top (numerator), using a decimal point to show tenths (one place), hundredths (two places), thousandths (three places), and so on.

Example 1: Change 3/8 to a decimal.
Write 3 ÷8. Divide, adding zeros as needed, until there is no remainder.

```
   .3 7 5
8) 3. 0 0 0
   2 4
     6 0
     5 6
       4 0
       4 0
```

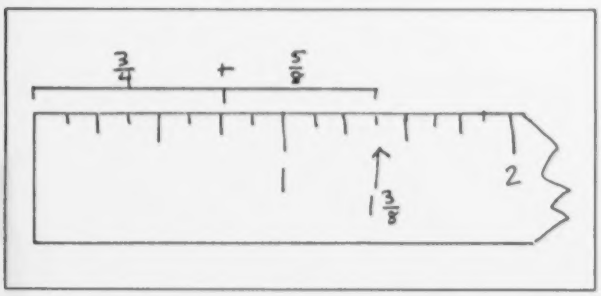
3/8 is equal to .375

Example 2: Change 3/7 to a decimal to the nearest hundredth.
Hundreths have two decimal places. Divide 7 into 3 and carry out the division to the third decimal place. Then round off to two places.

```
   .4 2 8
7) 3.0 0 0
   2 8
     2 0
     1 4
       6 0
       5 6
         4
```

3/7 is about equal to .43

Adding and Subtracting halves, quarters, and eighths can be done on a ruler.



4. RATIOS

Fractions **describe** parts while ratios **compare** parts.

Example: If there are 3 children and 2 adults in a family of 5, then

The **fraction** of family members who are children is 3/5.

The **fraction** of family members who are adults is 2/5.

The **ratio** of children to adults is 3 to 2.

Ratios are often written as fractions or by using a ":" sign. In the example above, the ratio of children to adults might be written as 3/2 or 3:2.

5. PERCENTS

Common Percents:
1/100 = 1% 1/10 = 10% 1/5 = 20%
1/4 = 25% 1/2 = 50% 1/3 = 33.3%

Finding a Percent of a Number:
There are several methods of doing this, as shown in the examples.

Example: What is 8% of 250?

Method 1) Multiply the number (250) by the percent (8) and divide by 100.
250 x 8 = 2000 2000 ÷ 100 = 20
20 is 8% of 250

Method 2) Divide the percent by 100, then multiply by the number.
8 ÷ 100 = .08 250 x .08 =20

Method 3) Find 1% of the number by dividing by 100. Then multiply by the desired percent.
1% of 250 = 2.5 2.5 x 8 = 20

Computing What Percent One Number is of a Second Number:

Write the first number over the second number and change that fraction to a percent. If it's not one of the common percents which you already know, change the fraction to a decimal (see above) and multiply the result by 100.

Example: What percent of your time do you sleep?

I sleep 6 hours in a 24-hour day.
6 hours is 6/24 of a day.
6/24 is 1/4 or 25%.

I sleep 7 hours in a 24-hour day.
7 hours is 7/24 of a day.
7/24 is about .292.
.292 x 100 = 29.2
7/24 is about 29.2%

Finding by What Percent Something Has Increased or Decreased:

Subtract the smaller number from the larger number.
Multiply the result by 100.
Divide by the starting number.

Example 1: By what percentage has your weight *increased* since birth?

Weight at birth: 8 lbs. (starting weight)
Weight now: 44 lbs.

44 — 8 = 36 36 x 100 = 3600
3600 ÷ 8 = 450
My weight has increased 450%.

Example 2: By what percentage has your weight *decreased* since you started to diet?

Weight before diet: 136 lbs. (starting weight)
Weight after six months on diet: 112 lbs

136 — 112 = 24 24 x 100 = 2400
2400 ÷ 136 = 17.65
My weight has decreased almost 18%.

APPENDIX

1. BOOKS ON MATH

There are many delightful children's stories with mathematical themes, engaging books about mathematics and books which may interest you as a parent. These are some personal favorites. Books which are starred are no longer in print, but you may find them on the shelves at your library.

COUNTING BOOKS

Anno's Counting Book by Mitsumasa Anno

Anno's Counting House by Mitsumasa Anno

Count and See by Tana Hoban

More than One by Tana Hoban

My Aunt and the Animals by Elizabeth MacDonald

One to Ten, Count Again* by Elaine Livermore

Over in the Meadow by John Langstaff and Feodor Rojankovsky

Ten, Nine, Eight by Molly Bang

Thirteen by Remy Charlip and Jerry Joyner

Trucks You Can Count On by Doug Magee

When Sheep Cannot Sleep by Satoshi Kitamura

When We Went to the Park by Shirley Hughes

SHAPES AND SIZES

Circles, Triangles, and Squares by Tana Hoban

Is It Larger? Is It Smaller? by Tana Hoban

Listen to a Shape by Marcia Brown

Look at Annette by Marion Walter

Make a Bigger Puddle, Make a Smaller Worm by Marion Walter

The Sesame Street Book of Shapes

STORY BOOKS

Alexander, Who Used to Be Rich Last Sunday by Judith Viorst

Bunches and Bunches of Bunnies by Louise Mathews

Cluck One by Louise Mathews

The Five Chinese Brothers by Claire H. Bishop

The 500 Hats of Bartholomew Cubbins by Dr. Seuss

A Grain of Rice by Helena Clare Pittman

How Big is a Foot? by Rolf Myller

Inch by Inch by Leo Lionni

Millions of Cats by Wanda Gag

Much Bigger Than Martin by Steven Kellog

One More Thing, Dad by Susan L. Thompson

The Sesame Street 1, 2, 3 Storybook: Stories About the Numbers from One to Ten by Emily P. Kingsley

The Shopping Basket by John Burningham

Six Foolish Fishermen by Benjamin Elkin

The Very Hungry Caterpillar by Eric Carle

BOOKS FOR OLDER CHILDREN

An Adventure in Geometry* by Anthony Ravielli

Anno's Math Games by Mitsumasa Anno

Anno's Mysterious Multiplying Jar by Mitsumasa Anno

Archimedes* by Martin Gardner

The Book of Think by Marilyn Burns

Everyday Weather and How It Works* by Herman Schneider

From Fingers to Computers* by Margaret F. Willerding

The Giant Golden Book of Mathematics* by Irving Adler

The Guiness Book of World Records

How Much is a Million? by David M. Schwartz

The I Hate Mathematics Book by Marilyn Burns

Mathematics, Magic, and Mystery by Martin Gardner

Nature Takes Shape by Dorothy and John Paull

People by Peter Spier

The Phantom Tollbooth by Norton Juster

Pyramid by David Macaulay

Shadows: Here, There, and Everywhere by Ron and Nancy Goor

****Shapes*** by Jeanne Bendick

Socrates and the Three Little Pigs by Tuyosi Mori

String, Straightedge, and Shadow by Julia E. Diggins

****Symmetry*** by Arthur G. Razzell and K.G.O. Watts

This Book is About Time by Marilyn Burns

****The Wonderful World of Mathematics*** by Lancelot Hogben

Yes-No; Stop-Go: Some Patterns in Mathematical Logic by Judith L. Gersting and Joseph E. Kuczkowski

Julie King

2. MATHEMATICAL GIFTS AND GAMES

MEASURING INSTRUMENTS

Measuring tape with both metric and English units
Folding carpenter's ruler
Measuring cups and spoons (good sets)
Watch
Stopwatch
Alarm clock
Kitchen Timer
Hand Tally Counter (Office Supply Store)
Calculator
Thermometer
Postal and/or kitchen scale
Fish scale
Rain gauge
Windspeed gauge
Compass (directional)
Pedometer
Odometer or speedometer for bicycle
Altimeter

COMMERCIAL GAMES

Battleship
Checkers
Chinese Checkers
Chess
Mastermind
Memory
Merlin
Monopoly
Payday
Simon
'Smath
Sorry
Tangrams
Yahtzee
Nearly any card game or board game with dice

TOYS

Nesting Cups
Stacking Rings
Tinkertoys
Lego (and other similar construction toys)
Wooden building blocks
Spirograph

OTHER GIFTS

Cookie cutters (including a rolling cutter)
Rubber stamps and stamp pad
Shape stickers and stencils
Bingo or Poker Chips
Playing Cards
Dice (can be homemade foam or wooden cubes)
Kaleidoscope
Dollhouse and furniture
Pegboard and pegs
Mirrors (unbreakable) for reflection, symmetry, angles and so on.
Play Money

3. RESOURCES

SOURCES OF INFORMATION

Most of the following sources of information are mentioned in the text and are listed here for your convenience. Many are free of charge, others can be obtained quite inexpensively. It's nice to have them handy.

Almanacs
Calendars and Date Books
Telephone Books
Postage rate information pamphlet from the post office
Zip code book
Public Service meter reading cards
Colorado Driver's Manual (for illustrations of road signs)
Maps
Catalogs
Restaurant menus
Sports and weather statistics from newspaper
Television schedules
Time tables

A MATH CORNER

It's also useful to have a variety of materials on hand that children may need for their mathematical explorations. If you have the space, you could keep these materials in a box, basket or on some empty shelves. This is a starter list to which you and your kids will probably want to add.

Pencils, colored markers, crayons
Erasors
Scotch tape, masking tape
String, colored yarn
Scissors, stapler, hole puncher
Rulers
Plain paper, graph paper, adding machine tape
Assorted buttons and other things for sorting
Toothpicks, drinking straws, popsicle sticks for measuring lengths
Marbles, beans, pebbles, washers for weighing
Spinners and markers from discarded games
and more...

4. A FORMULA TO FIND THE DAY OF THE WEEK[1]

To find the day of the week of any date between January 1st, 1800, and December 31st, 1999.

What day of the week was August 27th, 1915?

1. Take the last two digits of the year	15
2. Add a quarter of this, neglecting remainder	+3
3. Add the date of the month	+27
4. For January add 1 (if leap year add 0) For February add 4 (if leap year add 3) For March add 4 For April add 0 For May add 2 For June add 5 For July add 0 For August add 3 For September add 6 For October add 1 For November add 4 For December add 6	+3
5. For 19th century add 2 For 20th century add 0	+ 0 48

6. Divide by 7, and the remainder gives the day of the week

$48 \div 7 = 42$
Remainder 6

Sunday 1, Monday 2, Tuesday 3, Wednesday 4, Thursday 5, Friday 6, Saturday 0

August 27th, 1915 was a Friday.

---

[1]From ***Think of a Number*** Book I by G.A. Moss, Basil Blackwell, Oxford, 1958

5. INTUITIVE GEOMETRY[1] In Pre-school Education

In pre-school, children are encouraged to explore three-dimensional space. During the ensuing school years, their activities are restricted more and more:

to plane surfaces:	desktops, papers, pictures
to lines:	line up, write on the line
to a point in space:	sit still, stay put, don't get out of your seat...

But in pre-school education the environment is rich in geometric experiences:

SPHERE	balls, marbles, bubbles, beads...
CYLINDER	to climb through, oatmeal boxes, cans, pipes, straws...
CIRCLE	for playground games, dishes, rings, wheels...
ARC	when you swing, when you open a door, when you kick...
ANGLE	corners, knees, elbows, noses, heels, the hands of a clock...
CUBE	blocks, some jungle gyms, sugar cubes, some boxes...
RECTANGLE	doors, walls, ceilings, floors, windows, books, a sandbox...
TRIANGLE	to climb on, to slide down, to eat in a sandwich, to play in band...
CONE	for ice cream, funnels, megaphones, pine cones, spinning tops...
SQUARE	floor tiles, checkerboards, sidewalk squares...
TORUS	doughnut, tire, life saver, cheerio, poolside toy...

There's so much to touch, to pull, to swing, to crawl through and climb over in pre-school!

6. PALINDROMES[1]

A palindrome is a number that reads the same forward and backward such as 33, 868, 6006, or 52825.

423 is not a palindrome - BUT with a little addition 423 can be transformed into a palindrome.

```
  4 2 3
+ 3 2 4
  7 4 7      A palindrome!
```

We have written 423 backward and added it.

We call 423 a 1-step palindrome because we can turn it into a palindrome in 1 step.

Some numbers take longer:

```
          5 9
         +9 5
Step 1   1 5 4
         4 5 1     59 is a 3-step palindrome
Step 2   6 0 5
         5 0 6
Step 3   1 1 1 1
```

Choose some numbers of your own and find out out how many steps it takes to make each of your numbers palindromes.

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[1]Reproduced with permission from ***Pacific Oaks Leadership Development Institute*** leaflet. Pacific Oaks College, Pasadena, CA 91103.

[1]Reproduced with permission from ***Family Math*** by Jean Kerr Stenmark, Virginia Thompson and Ruth Cossey. Lawrence Hall of Science, University of California, Berkeley, CA 94720, p. 214.

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